ABSTRACT

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This thesis develops a method for optimizing the construction phases for rail transit extension projects with the objective of maximizing net present worth and examines the economic feasibility of such extension projects under different financial constraints (i.e. unconstrained, revenue-constrained and budget-constrained cases). A Simulated Annealing Algorithm is used for solving this problem.

A rail transit project is often divided into several phases due to its huge capital costs. A model is developed to optimize these phases for a simple, one-route rail transit system, running from a Central Business District (CBD) to a suburban area. The most interesting result indicates that the economic feasibility of links with low demand is affected by the completion time of those links and their demand growth rate after extensions. Sensitivity analyses explore the effects of input parameters (i.e. interest rate, taxation ratio, and operators and users unit cost) on optimized results (i.e. construction phases and objective). These analyses contribute

useful guidelines for transportation planners and decision-makers in determining construction phases for rail transit extension projects.

PHASED DEVELOPMENT OF RAIL TRANSIT ROUTES

By

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Chapter 1: Introduction

1-1 Background

Project scheduling is an important component in project management. The project scheduling phase assigns a start time to each project with respect to some constraints, such as resources of equipment, materials and labor with project work tasks over time [Martinelli, 1993]. Good scheduling can reduce problems due to production bottlenecks, facilitate the timely procurement of necessary materials, and otherwise insure the completion of a project as soon as possible. In contrast, poor scheduling can result in considerable waste as labor and equipment wait for the availability of needed resources or the completion of preceding tasks. Two scheduling approaches are often used: resource-oriented and time-oriented scheduling [Hendrickson, 1989]. For resource-oriented scheduling, the focus is on using and scheduling particular resources in an effective fashion. For time-oriented scheduling, the emphasis is on determining the completion time of the project given the necessary precedence relationships among activities. Both approaches emphasize the perspectives of the private sector rather than the users. For public transportation planning, scheduling should consider effects on both operators and users. The economic feasibility should be evaluated from the whole system's point of view. In addition, as transportation projects influence social and economic development, the decision regarding transportation investment must not be made solely on the basis of any single criterion. For example, the planners prefer not to overextend facilities so that the system have enough stations with high utilization rates, while the politicians want a route that appears to serve as many areas as possible. Transit operators want to maximize their profits or minimize their deficits. It is important to note that, generally, the capital investment costs of transportation projects are high, and incorrect investment decisions lead to misallocation of resources and money. Therefore, decisions must be carefully considered.

Figure 1.1 shows the FTA's required process for a new project. Various steps have to be considered by planners and decision-makers, including evaluation of different alternatives, preliminary engineering, environmental, traffic and economic impact studies. When a project has the approval of the government and goes to construction phase, the contractors usually prepare their construction schedules. The contractors' objective (cost minimization or profit maximization) may conflict with decision-makers' objective. For a high capital cost project, small changes in schedule could affect its benefits significantly. Consequently, a comprehensive analysis of economic feasibility and construction schedule is important for transportation projects.

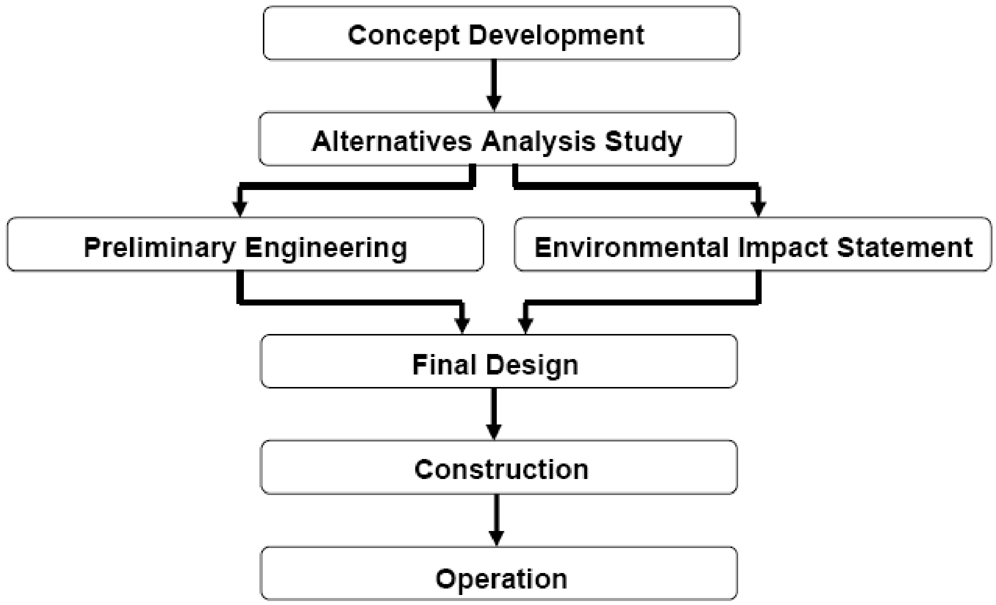

Fig. 1.1 FTA's Required New Starts Process (Source: FTA)

1-2 Problem Statement

A rail transit project has a significant construction cost, and may be uneconomical to build at one time. Therefore, it is often divided into several phases. Any addition of stations or extension of rail routes always affects many users and involves substantial investments. Figure 1.2 shows the structure for evaluation of new station additions to an existing rail transit route. Many consequences result from adding stations, including increased mobility, higher land value, increased employment opportunities, environmental impacts and reduced congestion. Therefore, such a project requires a comprehensive evaluation of all direct and indirect consequences, including positive and negative effects on different affected groups [Vuchic, 2005].

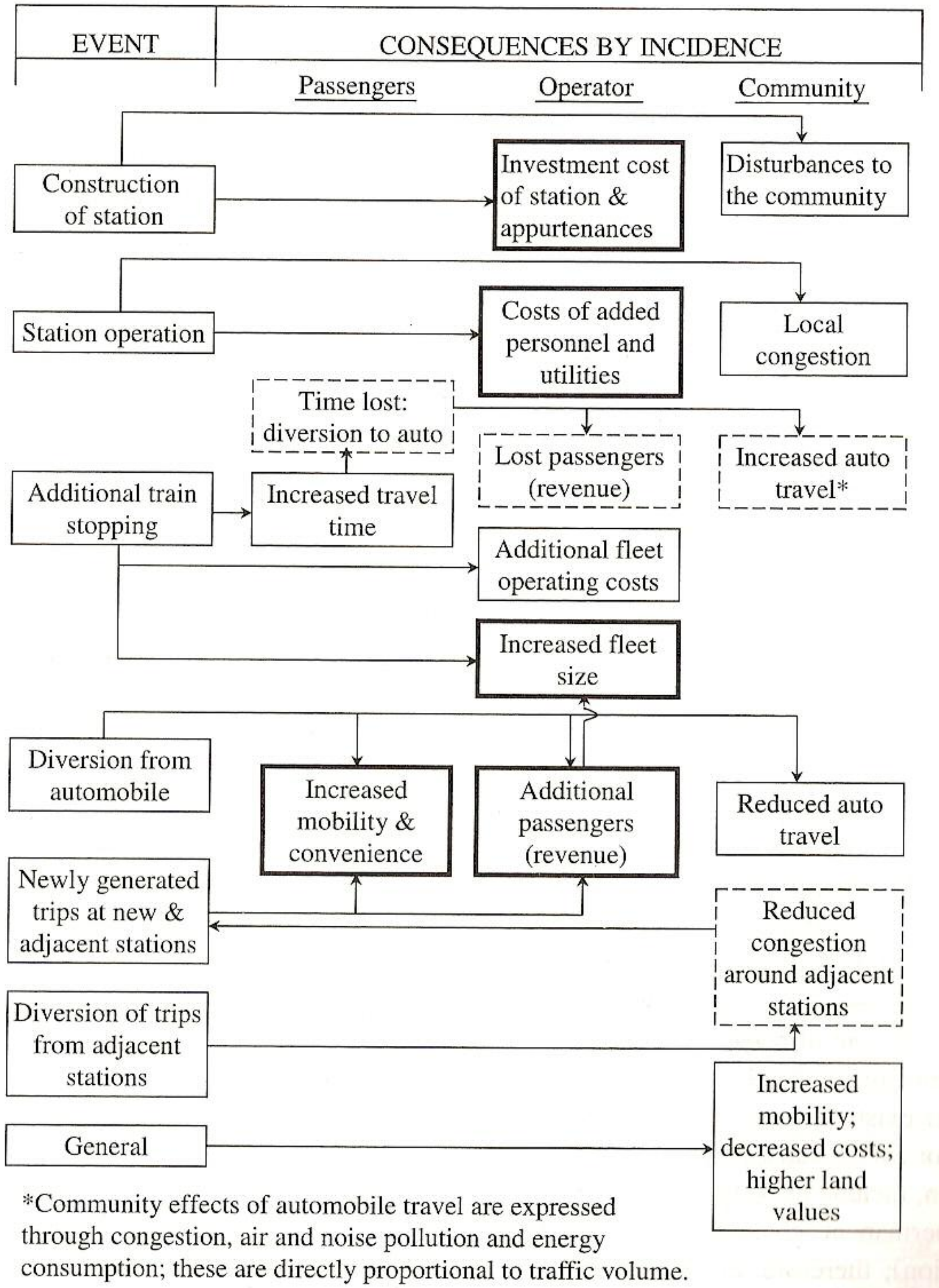

Fig. 1.2 Effects of Rapid Transit Line Extension (Source: Vuchic)

No general guidelines are yet available on how many phases are needed and when each phase should be implemented. The phases and execution time are usually based on available budgets, demand forecast, or probably some political reasons. The scheduled phases are probably not economically beneficial because of significant effects of extensions, such as that the travel demand tends to increase faster after a better transportation service is available and more potential demand is generated by adding new stations. Any decision affects the future results for the entire analysis period. Therefore, we are proposing a method to determine when is the best time to implement extensions and how many phases we should have for a given route and planning horizon. Project evaluation and scheduling are performed simultaneously. The solution will be an indicator of the desirability of a project from the standpoint of a decision maker. Based on different constraints incorporated into the model, not only the phased development plan but also the entire financial plan and operational plan can be determined through the model.

1-3 Objective

The objective of this study is to determine when and how to extend a transit route in order to optimize overall net benefits. Since demand and benefits may be significantly affected by adding additional stations, all quantifiable items must be computed on a life-cycle basis. Since optimizing the system merely based on total cost always ends up with doing nothing, it cannot be used to evaluate different alternatives. Therefore, the construction phases are optimized by maximizing the net present worth of total benefits for the entire analysis period.

1-4 Organization

This thesis is organized as follows:

Chapter 2 first reviews the theoretical and empirical literature on models for rail transit systems that seek to optimize total cost and total net benefits. Then, this chapter reviews other scheduling problems which use heuristic approaches to get near-optimal solutions. In addition, it considers the performances of different heuristic approaches. Based on such comparison, the Simulated Annealing Algorithm is adopted for this study.

Chapter 3 develops an integer programming optimization model for evaluating transit extension projects with various financial constraints.

Chapter 4 presents the methodology for solving the proposed mathematical model and discusses the tuning of its parameters. The design of the neighborhood structure and the choice of the parameter values are addressed in this chapter.

Chapter 5 presents numerical examples and demonstrates the performance of the proposed model. The system evaluation shows the optimized solutions that maximize the net present based on given input parameters, and compares these results obtained with various constraints.

Chapter 6 presents the sensitivity analysis for several major input parameters. The sensitivity analysis investigates how the optimized results are affected by changes in input parameters.

Finally, chapter 7 summarizes the research findings and implications of this study. Future research directions are then proposed.

Chapter 2: Literature Review

Chapter 2 summarizes previous studies related to this thesis. The literature reviewed in this section is divided into the following categories: (i) scheduling problems, (ii) transit optimization models, (iii) comparisons of heuristic approaches.

2-1 Scheduling Problems

Scheduling problems determine optimal schedules under various objectives, different constraints and characteristics of the systems. This thesis considers a rail transit extension project scheduling problem whose objective function is maximizing the net present worth, taking into account the funding availability. Numerous studies can be found about scheduling transit crews, timetable and maintenance activities. However, no previous studies about rail transit extension have been found. The key words used in the search process include rail transit extensions, phased development and transit segmental analysis. Also all available resources are exhausted. There must be models or criteria used by consultants and contractors, but probably not published in scientific journals. Nonetheless, this problem can be treated as a resource-constrained project scheduling problem (RCPSP).

Kolisch and Padman (2001) summarized and classified previous studies on the RCPSP by their objectives and constraints: net present value (NPW) maximization and makespan (defined as the total duration of a project) minimization, with and without resource constraints. Numerous studies are surveyed with a perspective that integrates models, data, and optimal and heuristic algorithms, for the classes of project scheduling problems. Here, only topics related to NPW maximization are reviewed. This paper shows that when substantial cash flows are present in the project, in the form of expenses for initiating activities and progress payments for completion of parts of the project, the net present value (NPW) criterion is a more suitable measure of project performance than others. Many methods are used to solve this scheduling problem. Calculus, enumerative search, mathematical programming, branch and bound, and other problem-dependent algorithms can be used only if the problem is sufficiently small or well behaved. For large and complex problems, heuristic algorithms are often applied to determine solutions that are close to the global optimum. Tabu search, Genetic Algorithms, and Simulated Annealing Algorithms are commonly used in previous studies. The paper [Kolisch et al., 2001] shows that problem-independent, metaheuristic approaches are better able to exploit the complex interactions of many critical parameters of RCPSP in comparison to the single-pass, parameter-based, and problem-dependent heuristics. Kolisch and Padman (2001) also summarize useful results for RCPSP when maximizing NPW. For the resource-unconstrained case, generally it is optimal to schedule jobs with associated positive cash flows as early as possible, and jobs with net negative cash flows as late as possible subject to restrictions imposed by network structure. For the resource constrained case, at high cost of capital or long project duration, it is important to evaluate bonus/penalty and capital constraints when scheduling activities.

Ahern et al. (2006) developed a multi-objective investment-planning model to determine priorities of different railway projects. Both qualitative and quantitative criteria are considered in the model. Several attributes that affect investment decision-making were identified and estimated by questionnaire, and weights are given to these attributes. The results shows that user benefits are the most important element in investment decision-making, followed by safety/accident benefits and the total economics benefits of the project. NPW is rated to be the second least important among the attributes considered in this survey in railway selection. Table 2.1 shows the estimated weightings of the attributes in railway project prioritization for investment. However, there are some weak points in the model. First, although the model is multi-objective, the investment decisions are made with the objective of optimizing each attribute one by one. After that, average weighting values are applied to get the final decision. Optimizing some attributes conflicts with optimizing others. For instance, if the objective is to minimize capital costs, the other objective that maximizes passengers on train cannot be achieved. A promising algorithm or method should be used to solve this problem. In addition, it's difficult to quantify qualitative items. Detailed methods for calculating those quantitative attributes are not shown in this paper. Third, if all the attributes, both quantitative and qualitative items, can be estimated correctly, using NPW as criterion is feasible for evaluating all the options. Here, NPW is defined as discounted benefits minus discounted costs. Although this has some drawbacks, it still indicates that the important attributes (user benefits, capital costs, and economic benefits) should be considered in railway projects.

Table 2.1 Estimated Weightings of Attributes for Railway Project Selection (Source: Ahern)

Attribute/Goal	Weight
User benefits	0.092
Safety/accident benefits	0.091
Total economic benefits	0.088
Capital cost	0.085
To support land use, social and economic policy at local, national and regional level	0.079
Additional passengers on train	0.078
Benefit/cost ratio	0.078
To exploit the particular strengths of rail to provide a highly integrated and competitive public transport service	0.076
Car resource cost saving	0.073
To improve environmental quality and health	0.073
Increase in revenue in railway	0.067
Net present value	0.062
To promote sound project selection measures	0.057

Valadares Tavares (1987) optimizes the schedule for a set of interconnected railway projects with the purpose of maximizing its total net present value, using Dynamic Programming. This model is applicable to schedule large sets of expensive and interconnected development projects under tight capital constraints and with a marginal net present value. He notes that maximizing the NPW of a project in terms of its schedule under eventual restrictions concerning its total duration can be considered as a dual perspective of the problem of minimizing makespan with resource constraints. The model presented in the paper does not consider the effects of interrupted demand when project is undergoing. The items considered in NPW are only construction expenditures and payments received after completion of projects.

Since it is a renewal project, all the items that are affected by the project should be taken into account.

Wang and Schonfeld (2007) develop a simulation model to evaluate waterway system performance and optimize the improvement project decisions with demand model incorporated. They maximize the present worth of net benefits for the entire analysis period rather than minimize total costs, since traffic demand and benefits are significantly affected by the simulated decisions. Different scenarios are tested (with and without lock capacity reductions during work closure periods or with and without demand elasticity). The results show that more negative demand elasticity with respect to travel time can significantly reduce traffic during work closures. If considering a renewal project, demand elasticity is a main factor and it should be considered in the model. In this thesis, the extensions will not affect the current users in the network at all, so the demand elasticity can be omitted in this problem.

2-2 Transit Optimization Models

This section reviews relevant studies on transit optimization models.

Matisziw et al. (2006) proposed an optimization model to determine the route extension network for bus transit systems. It is similar to a routing problem with the objective that maximizes covering areas and minimizes the extension length under resource constraints. It is important to expand the existing service network to tap into emerging areas of demand not being served. Maximizing network coverage can increase ridership. While increasing this potential ridership is significant, it is necessary to keep any route extension to a minimal length, since extending the route to low demand areas could result in low service utilization. That is why the bi-objective model is used to avoid overextending an agency's existing facilities. This problem only determines the network, rather than extension phases. It can be seen as a preliminary analysis of the phased development problem. In addition, the NPW maximization objective used in this thesis can also prevent overextending the facilities.

Basically, the approach to modeling and design the transit system which is used in this thesis is based on the work of Chien and Schonfeld (1998), except for the decision variables. Chien and Schonfeld (1998) developed a joint optimization model that optimizes the characteristics of a rail transit route and its associated feeder bus routes considering minimizing total costs. Spasovic and Schonfeld (2003) optimized the transit service coverage with the objective that minimizes total costs. The results show that in order to minimize total costs, the operator cost, user access cost, and user wait cost should be equalized. It is noted that the most significant factor in determining the rail line length is the demand. Since the demand is the main factor in determining the transit line length, no completion constraint is considered in the model. Consideration of the completion constraint may result in overextending the facilities.

The most common objective functions are minimizing total costs, maximizing profits and welfare. Numerous previous studies focus on optimizing transit operational and design characteristics. However, papers about optimizing construction phasing for rail transit have not been found. The papers listed above show how a transit system can be modelled and what variables should be considered.

2-3 Heuristic Approach Comparison

Heuristic approaches are widely used in scheduling problems, since they are more efficient in finding a near-optimal solution for complex problems. Sechen and Sangiovanni-Vincentelli (1985) developed a computer package based on Simulated Annealing to deal with circuit placement and wiring problems. Golden and Skiscim (1986) used SA to solve routing and location problems. Wilhelm and Ward (1987) applied Simulated Annealing to solve quadratic assignment problems. Martinelli and Schonfeld (1993) introduced a heuristic technique for the sequencing and scheduling of the inland waterway lock improvement projects. Bouleiemn and Lecocq (2003) used modified Simulated Annealing for the resource-constrained project scheduling problem and its multiple mode version. Wang and Schonfeld (2006) developed a simulation-based optimization model for selecting and scheduling waterway improvement projects by using Genetic Algorithm.

Hasan et al. (2000) tested several metaheuristic approaches (i.e. simulated annealing, genetic algorithm, and tabu search) for the unconstrained quadratic Pseudo-Boolean function. Several parameters are tested and identified to observe their performances in terms of solution quality and computation time. The results show that GA performs well compared to other algorithms. Tabu Search (TS) seems to have failed in obtaining competitive solutions and running one the test problems. Arostegui et al. (2006) compared the relative performance of Tabu Search, Simulated

Annealing (SA) and Genetic Algorithms (GA) on several types of Facility Location Problems (FLP) considering time-limited, solution-limited and unrestricted conditions. The solutions show that overall TS has the best performance, followed by SA and GA. Wang et al. (2006) compared Simulated Annealing (SA) and Genetic Algorithms (GA) for two/three-machine no-wait flow problems. From the example problems, it was found that SA is superior to GA in both solution quality and computation efficiency under identical termination criteria.

The three papers listed above show that the performance of heuristics varies in different kinds of problems. It is important to note that the parameters of these heuristics compared in these papers may affect its results and the conclusions. In addition, the skills and experience of the users with these tools also influence performance. Even though same parameters are identified for different methods, there are some parameters that are not identifiable due to the structure of each heuristic.

From all the examples dealing with scheduling problems, Simulated Annealing is selected for use in the thesis because of its simple concept, relative ease of implementation and ability to provide reasonably good solutions for many combinatorial problems. In the chapter 4, some important parameters and tuning techniques for SA are discussed.

2-4 Summary

As reviewed above, previous studies about rail transit extension scheduling are scarce, but this problem can be treated as a resource-constrained project scheduling problem (RCPSP) with unique characteristics. First, the activities in this project represent the stations to be added. Second, the precedence relations in this problem are much easier than in the general PSP. The transit route can only be extended sequentially from one end (i.e. CBD) to the other. Third, constraints on two resources are considered in this thesis: capital budget and revenue. For the capital budget constraint, subsidies are divided into equal parts for each time interval. The revenue constraint is used for balancing the operational expenditure. It is important to note that the resource constraints vary over the entire time horizon, since these two constraints are affected by operational situation and the decision made in the previous year. Hence, this problem is dynamic RCPSP. Maximization of the net present worth is the objective. All the quantifiable items that would be affected by the extension should be considered in this problem (e.g. user waiting costs, in-vehicle costs, operating and maintenance costs), including socio-economical effects if they can be quantified and estimated correctly. Due to the complexity of the dynamic RCPSP, Simulated Annealing is applied to solve this problem. Detailed design of SA and parameter tuning will be discussed in Chapter 4.

Chapter 3: Model Formulation

In this chapter, an integer programming model is formulated to evaluate the decisions. In addition, different financial constraints are tested. To simplify notation, the following analysis expresses benefit and cost functions as if only one time interval is analyzed. We repeat the analysis for every time interval and then sum them up. Table 3.1 defines the notation for variables that will be used in the thesis.

Table 3.1 Notation

Variables	Descriptions	Units
C_C	Capital cost	$
C_I	In-vehicle cost	$
C_M	Maintenance cost	$
C_{OR}	Operating cost	$
C_S	Supplier cost	$
C_U	User cost	$
C_W	Waiting cost	$
d	Station spacing	mile
F_T	Fleet size	vehicle
h	Headway	hour
i	The origin in the O/D matrix	-
j	The destination in the O/D matrix	-
k	Capital cost for station and rail line	$
m	The row in the O/D matrix	-
n_C	Number of cars needed per train	cars/vehicle
P	Fare price	$
NPW	Net present worth of total benefits	$

q_{ij}	Traffic flow from origin i to destination j	people
Q	Demand function	-
r	Demand growth rate	-
R	Round trip time	hour
s	Interest rate	-
t	Time interval	-
t_d	Dwell time	hour
TB	Total benefit	$
TC	Total cost	$
TNB	Total net benefit	$
u_I	Unit cost of user in-vehicle time	$/passenger-hr
u_L	Maintenance unit cost	$/passenger-mile
u_T	Hourly operating cost	$/vehicle-hr
u_W	Unit cost of user waiting time	$/passenger-hr
U_B	User benefit	$
V	Cruise speed	miles/hr
y	Decision variable	-

3-1 Model Assumptions

In order to simplify the problem, the following assumptions are made:

1. A given demand at the starting time interval ($t = 0$) is already consistent with network equilibrium.
2. Transit routes and station locations are predetermined so that the user access costs can be omitted.
3. Effects of development schedules of other routes on the demand of our route are neglected.
4. Stations can only be added sequentially from the CBD to the rural area.

5. There are no binding construction time constraints.
6. Potential demand for each O/D pair increases at a higher rate after the station is completed.
7. Capital costs are discounted if multiple links are built at one time (in the same year).
8. The interest rates are effective interest rates which already consider inflation rates so that we need not to transform the cash flow from actual dollars to constant dollars.

Figure 3.1 shows the proposed route. The proposed transit system is 54.4 miles long with 30 stations. Currently only 4 stations are completed and in service. The study time horizon is 30 years. Our decision variable $y_i^{(t)}$ is having links and stations or not. $y_i^{(t)} = 1$ represents that link i exists in time period t; $y_i^{(t)} = 0$ represents that link i has not been built in time period t. Here link i is defined as the section between station $i-1$ and i, and link i includes station i. $y_5^{(2)} = 1$ represents that link 5 is added in year 2.

Decision Variable: $y_i^{(t)} = 0$ or 1, $i = 1, 2, \ldots,$ $t = 0, 1, 2, \ldots$

i denotes links, and t denotes time interval.

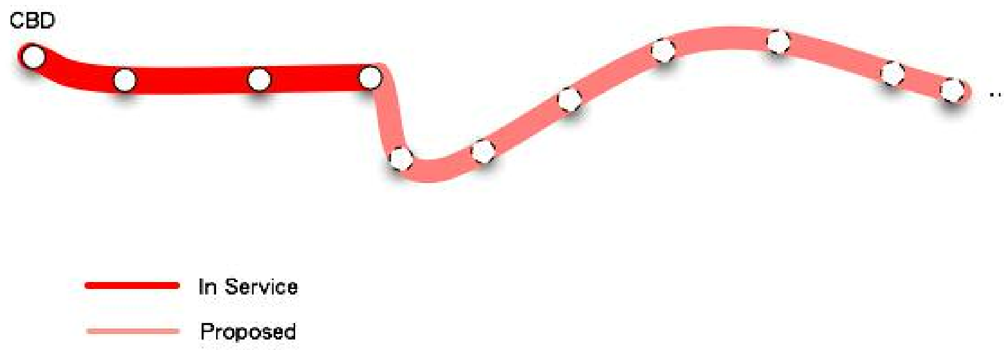

Fig. 3.1 Proposed Route

In the long term, the traffic increase may occur due to demographic and economic growth. Demand growth is considered here by multiplying the demand elasticity relation with a compound growth rate $(1+r)^t$, where r is the growth rate per time interval (e.g., per week, month or year) and t represents intervals (e.g., weeks, months or years) of growth.

As discussed above, the origin/destination (O/D) matrix values can continuously increase at a specific annual growth rate based on traffic demand forecasts.

$q_{ij}^{(t)} = q_{ij}^{(0)} * (1+r)^t$, $\forall i, j$, where q_{ij} denotes traffic flow from origin i to destination j. O/D matrix is symmetric, where $q_{ij} = q_{ji}$. There are 4 stations in service at time interval zero. The O/D matrix is

$$OD^{(t)} = \begin{bmatrix} - & y_2 q_{12} & y_3 q_{13} & y_4 q_{14} & y_5 q_{15} & y_6 q_{16} & \cdots \\ y_2 q_{21} & - & y_3 q_{23} & y_4 q_{24} & y_5 q_{25} & y_6 q_{26} & \cdots \\ y_3 q_{31} & y_3 q_{32} & - & y_4 q_{34} & y_5 q_{35} & y_6 q_{36} & \cdots \\ y_4 q_{41} & y_4 q_{42} & y_4 q_{43} & - & y_5 q_{45} & y_6 q_{46} & \cdots \\ y_5 q_{51} & y_5 q_{52} & y_5 q_{53} & y_5 q_{54} & - & y_6 q_{56} & \cdots \\ y_6 q_{61} & \cdots & \cdots & \cdots & \cdots & - & \cdots \\ y_7 q_{71} & \cdots & \cdots & \cdots & \cdots & \cdots & - \end{bmatrix}^{(t)}$$

, where at $t = 0$, $y_1 = y_2 = y_3 = y_4 = 1$, $y_5 = y_6 = ... = 0$.

3-2 Benefit Function

3-2-1 User Benefit

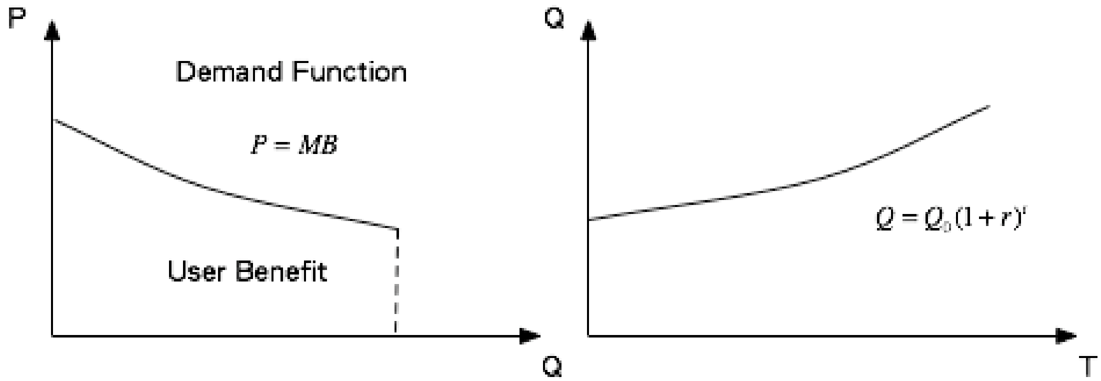

Fig. 3.2 User Benefits

User benefit (U_B), in any time interval t, is defined as the area under the demand (= marginal user benefit) curve for that interval, integrated from 0 to $q_{ij}^{(t)}$, where $q_{ij}^{(t)}$ is the traffic flow from i to j in the t^{th} simulation interval. Since q_{ij} may fluctuate in different intervals, then the overall user benefit for the entire analysis period is

$$U_B = \sum_t \sum_i \sum_j \left(\int_0^{q_{ij}^{(t)}} P \cdot dQ \right) \quad ,i \neq j \qquad (3\text{-}1)$$

3-3 Cost Function

Our total cost function consists of supplier cost and user cost, as discussed below.

3-3-1 User Cost

The user cost (C_U) consists three components: in-vehicle cost, waiting cost and access cost.

Access cost is the total demand multiplied the access time. Because we assume station locations are predetermined, the access cost might be omitted.

The waiting cost, C_W, is the total demand multiplied by the waiting time which is half of the headway, h, and the unit cost of user waiting time, u_W ($/passenger-hour):

$$C_W^{(t)} = OD^{(t)} * h/2 * u_W \tag{3-2}$$

In-vehicle cost, C_I, is the through flow multiplied by the in-vehicle time which includes the riding and dwell time and the unit cost of in-vehicle time, u_I ($/passenger-hour). Through flow is equal to inflow minus outflow at each station, as shown in Figure 3.3, and it can be formulated from the O/D matrix:

$$\begin{bmatrix} - & y_2q_{12} & y_3q_{13} & y_4q_{14} & y_5q_{15} & y_6q_{16} & \cdots \\ y_2q_{21} & - & y_3q_{23} & y_4q_{24} & y_5q_{25} & y_6q_{26} & \cdots \\ y_3q_{31} & y_3q_{32} & - & y_4q_{34} & y_5q_{35} & y_6q_{36} & \cdots \\ y_4q_{41} & y_4q_{42} & y_4q_{43} & - & y_5q_{45} & y_6q_{46} & \cdots \\ y_5q_{51} & y_5q_{52} & y_5q_{53} & y_5q_{54} & - & y_6q_{56} & \cdots \\ y_6q_{61} & \cdots & \cdots & \cdots & \cdots & - & \cdots \\ y_7q_{71} & \cdots & \cdots & \cdots & \cdots & \cdots & - \end{bmatrix}^{(t)}$$

$$\text{Through Flow} = 2^* \sum_{m=1}\left[\sum_{i=1}^{m}\left(\sum_{j=i+1} y_j q_{ij} - \sum_{j=1}^{i} y_i q_{ij}\right)\right] \quad (3\text{-}3)$$

where m = the row in the O/D matrix

i = the origin in the O/D matrix

j = the destination in the O/D matrix

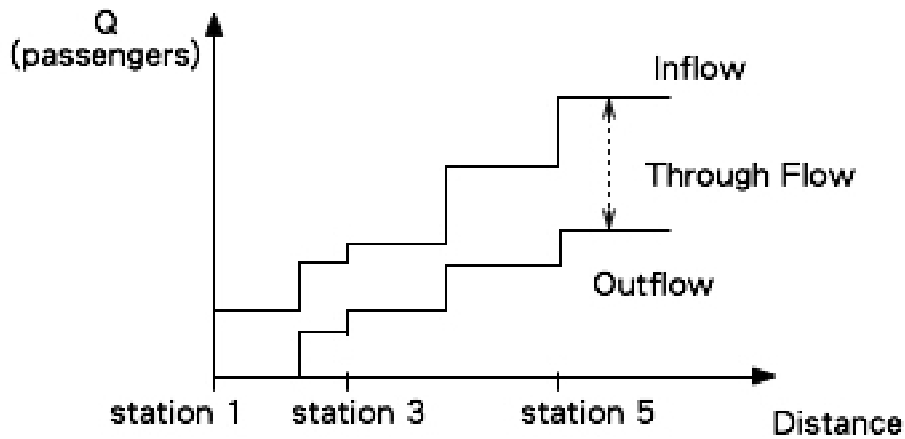

Fig. 3.3 Through Flow

$$C_I = 2 * \sum_{m=1}\left[\sum_{i=1}^{m}\left(\sum_{j=i+1} y_j q_{ij} - \sum_{j=1}^{i} y_i q_{ij}\right)\right] * \left(\frac{d_{m+1}}{V} + t_d\right) y_{m+1} * u_I \quad (3\text{-}4)$$

d_{m+1} represents the station spacing between station $m+1$ and m. V is the transit speed. t_d is the lost time at each station. The factor t_d accounts for the time lost through deceleration and acceleration as well as for dwell time at a station.

No out-of-pocket costs were included in the user cost. Transit fares are not part of the user cost because they are merely transfer payments from users to operators. Thus, the user cost (C_U) is equal to waiting cost plus in-vehicle cost:

$$C_U = C_W + C_I \tag{3-5}$$

3-3-2 Supplier Cost

The supplier cost (C_S) consists of three components shown in Equation 3-6:

$$C_S = C_C + C_{OR} + C_M \tag{3-6}$$

These are capital cost (C_C), operating cost (C_{OR}) and maintenance cost (C_M).

Capital cost (C_C) includes land acquisition, design and construction, and rail line pavement costs:

$$C_C = \sum_t \sum_i (y_i^{(t)} - y_i^{(t-1)}) k_i \tag{3-7}$$

where k_i is the fixed costs for link i. We use $y_i^{(t)} - y_i^{(t-1)}$, since k_i is the cost which only counts when we build link i in the first beginning.

There exist some economies if more than one station is built at one time, since the setup costs can be reduced. In the numerical examples of this study, the construction cost savings are set at 3% for 2 stations, 6% for 3 stations, 9% for 4 stations, 12% for 5 stations, 15% for 6 stations, 18% for 7 stations, 21% for 8 stations, 24% for 9 stations, and 24% for more than 10 stations.

The operating cost is the transit fleet size F_T multiplied by the hourly operating cost per car u_T (\$/vehicle-hour) and the number of cars n_C needed per train. Because the optimal headway will change as we extend the route, we have to update the headway after every decision made. To obtain the fleet size, the transit round trip time $R^{(t)}$ is derived first:

$$R^{(t)} = 2\sum_i \left(\frac{d_{i+1}}{V} + t_d \right) y_{i+1}^{(t)} \tag{3-8}$$

d_{i+1} represents the station spacing between station $i+1$ and i. Since the demand function is not elastic with respect to headway, the optimal headway h is found by checking the first order derivative of the total cost function with respect to h equal to zero and solving it for h. The second derivative of the total cost function with respect to h is also checked to make sure that the total cost function is a convex function. Minimizing operating costs is used to provide enough service while maximizing NPW.

$$\frac{\partial TC}{\partial h} = 0 \tag{3-9}$$

$$\frac{\partial TC}{\partial h^2} = \frac{2Rn_C u_T}{h^3} > 0 \tag{3-10}$$

The resulting optimal headway is

$$h^{(t)} = \sqrt{\frac{2n_C u_T y_{i+1}^{(t)} \sum_i \left(\frac{d_{i+1}}{V} + t_d \right)}{u_W \sum_i y_i^{(t)} q_{ij}^{(t)}}} \tag{3-11}$$

The fleet size $F^{(t)}$ is then the transit round trip time divided by the headway h:

$$F^{(t)} = \frac{R^{(t)}}{h^{(t)}} \tag{3-12}$$

Then we have the operating cost:

$$C_{OR}^{(t)} = F^{(t)} n_C u_T \tag{3-13}$$

Maintenance cost, C_M, is the passenger-miles traveled (PMT) multiplied the transit line unit cost, u_L (\$/pass. mile):

$$C_M = 2 * \sum_{m=1} \left[\sum_{i=1}^{m} \left(\sum_{j=i+1} y_j q_{ij} - \sum_{j=1}^{i} y_i q_{ij} \right) \right] * \left(\frac{d_{m+1}}{V} + t_d \right) y_{m+1} * u_L \quad (3\text{-}14)$$

Therefore, the supplier cost is equal to operating cost plus maintenance cost:

$$C_S = C_{OR} + C_M + C_C \quad (3\text{-}15)$$

The objective function is the system's net present worth (NPW). The net benefit for a period of time is equal to total benefit minus total cost. Total benefit includes user benefit B_U; total cost includes supplier cost C_S and user cost C_U.

$$TNB = TB - TC = B_U - \left(C_S + C_U \right) \quad (3\text{-}16)$$

Because money can earn a certain interest rate s through its investment, a dollar received/spent in the future is worth less than a dollar in hand at present. We have to include the interest rate in the model to obtain the net present worth.

$$NPW = \sum_t \frac{TNB}{(1+s)^t} \quad (3\text{-}17)$$

3-4 Proposed Optimization Model

Equations 3-18 to 3-23 present an optimization model that seeks to maximize the net present worth.

Maximize $\quad NPW = \sum_t \left(TB - TC \right) \left(1 + s \right)^{-t} \quad (3\text{-}18)$

Subject to $\quad y_i^{(t)} = 1 \text{ or } 0 \quad (3\text{-}19)$

$$y_i^{(t)} - y_i^{(t-1)} \geq 0 \text{, for all } i \text{, } t \geq 1 \tag{3-20}$$

$$y_i^{(t)} - y_{i+1}^{(t)} \geq 0 \text{, for all } t \text{, } i \geq 1 \tag{3-21}$$

$$0.7 * revenue^{(t)} \geq C_{OR}^{(t)} + C_M^{(t)} \text{, for all } t \tag{3-22}$$

$$0.3 * revenue^{(t-1)} + Subsidy^{(t)} \geq C_C^{(t)} \text{, for all } t \tag{3-23}$$

Equation 3-19 is the binary integer constraint for decision variables. Equation 3-20 is the realistic constraint that insures that after link i has been built, it will always stay in operation. Equation 3-21 is the precedence constraint that forces any link i not to start if any one of its predecessors in the set $\text{P}red_i$ has not ended. The transit route has to be built sequentially, since there would be fewer benefits if we randomly choose any segment to build along the route. In transit operation, some fraction of the fare collection is used for covering operation expenses, and the other fraction can be used for subsidizing the construction of new transit route extensions. Equation 3-22 is the revenue constraint for covering operational expenses, i.e. operating and maintenance costs. Due to the uncertainty about the future, the transit operators tend to balance their operation-related expenditures in each year. Thus, 70% of the fare collection is used for covering the operating and maintenance costs in each year. Equation 3-23 is the budget constraint for funding the capital investments. Assuming the federal government pays all the capital costs for extensions, the funds are divided into equal parts and available at the beginning of each year. Equation 3-23 shows that the construction costs have to be lower than capital funds plus 30% of the fare collection in the previous year.

Chapter 4: Methodology

There are several well-known methods for finding near-optimal solutions to linear and nonlinear optimization problems. These include Tabu Search and Genetic Algorithms. Simulated Annealing is one such heuristic optimization technique. It is based on annealing process to escape from local optimum to find a near-optimal solution. Simulated Annealing is similar to hill climbing or gradient search with some modifications. In gradient based search, the search direction depends on gradient and hence the objective function should be a continuous function without discontinuities. Simulated Annealing does not require the function to be smooth and continuous since it is not based on the function's gradient.

The best-known example for Simulated Annealing is the Traveling Salesman Problem (TSP). Given a list of N cities and a means of calculating the traveling costs (distances) between any two cities, one salesman must pass through all the cities one time and return to the original point. The objective is to minimize traveling costs (distances). The basic concept for Simulated Annealing is to search the neighborhood solution. If the neighborhood solution is better than the previous one, it is always accepted, then search possible neighborhood solutions again. To escape the problem of getting stuck in a local minimum occasionally routes with costs (distances) greater than the current route are also accepted but with a probability similar to the probability in the dynamics of the annealing process. As the temperature decreases, the probability of accepting a bad solution is decreased and in the final stages the Simulated Annealing algorithm becomes similar to gradient based search.

4-1 Parameters of Simulated Annealing

Kirkpatrick and Gelatt (1983) listed four needed components for using Simulated Annealing: (1) a concise description of a configuration of the system; (2) a random generator of "moves" or rearrangements of the elements in a configuration; (3) a quantitative objective function containing the trade-offs which have to be made; and (4) an annealing schedule of the temperatures and length of times for which the system is to be evolved. The random generator to move to the neighborhood solution and cooling schedule are the key components of good SA performance.

The sequences of temperature values are critical when implementing SA. Many methods have been proposed in literature to compute the initial temperature T_0. Kirkpatrick (1983) estimated $T_0 = \Delta E_{\max}$ where $\Delta E_{\max}$ is the maximal cost difference between any two neighborhood solutions. A more precise estimation is proposed with multiple variants by Aarts et al. (1997). $T_0 = K\sigma_\infty^2$ is recommended where K is a constant typically ranging from 5 to 10 and σ_∞^2 is the second moment of the energy distribution when the temperature is ∞. σ_∞ is estimated using a random generation of some solutions. Johnson et al. (1989) computed the initial temperature by using the formula $T_o = -\dfrac{\overline{\Delta E}}{\ln(x_0)}$, where $\overline{\Delta E}$ is an estimate of the cost increase of strictly positive transitions, x_0 is the initial acceptance ratio which is the number of accepted bad transitions divided by the number of attempted bad transitions. Triki et al. (2005) used the same formula to compute the initial temperature by setting x_0 = ½.

The cooling schedule is also very important to Simulated Annealing. Each problem requires a unique cooling schedule and it becomes very difficult to pick the most appropriate schedule within a few simulations. If the temperature decreases too quickly, then the algorithm can easily get stuck in a local optimum solution. A fast cooling schedule is similar to greedy algorithm. If the temperature decreases too slowly, then the algorithm requires more computation to achieve convergence [Cheh et al., 1991]. The most frequently used cooling decrement rule is $T_{k+1} = \alpha * T_k$, where α denotes the cooling factor. Usually α is within the range between 0.5 and 0.99. The advantage of using this cooling rule is very simple. Many other adaptive cooling schedules have been proposed to shorten the annealing process as possible. In adaptive cooling schedules, the next temperature value is based on the history temperature path. It's important to note that each problem has its own best cooling schedule. There is no particular cooling schedule that can guarantee the optimality or near-optimality of the annealing process for all kinds of problems.

The performance of SA strongly depends on the annealing parameters and the structure of the neighborhood search [Ben-Ameur, 2004]. These two elements are discussed in the next two sections.

4-2 Neighborhood Structure

In order to use SA Algorithm, there must be a random generator in the configuration, that is, a procedure for taking a random step from x to $x + \Delta x$. The solution configuration is introduced first, and then the neighborhood structure is presented. The solution vector is listed below:

$$x_0 = [4 \quad 4 \quad 5 \quad 7 \quad 7 \quad 7 \quad 7 \quad 9 \quad \ldots \quad 28]$$

The numbers in the vector represent number of stations in service in each year. $x_0(2) = 4$ represents that there are 4 stations in service in year 2. $x_0(7) = 7$ represents that there are 7 stations opened in year 7. $x_0(1)$ represents the current status at $t = 1$, with only four stations in service. We can easily know in which year to extend the transit route by the increments from the vector. To jump to the next neighborhood solution, the neighborhood generation performs as follows: randomly choose one number from $x_0(2)$ to $x_0(30)$. $x_0(1)$ is the initial status at $t = 1$, so we cannot change it. Then we randomly generate a number between -5 to 5. The number is the station increment or decrement. Finally, adjustments are made to make the solution feasible. One important characteristic in the solution configuration is that once a station has been built in a specific year, it cannot disappear in the following years. Therefore, if we make any changes in one year, we have to check the feasibility for the vector. For example, $x_0(2)$ is chosen, and the random number generated is 2. The $x_0(2)$ = 6 which indicates that six stations are in service in year 2. Then we check the feasibility. There are only 5 stations in $x_0(3)$, so we have to add one more station to make the entire solution feasible. The next neighborhood solution x_1 is

$$x_1 = [4 \quad 6 \quad 6 \quad 7 \quad 7 \quad 7 \quad 7 \quad 9 \quad \ldots \quad 28].$$

Another example in decrements, $x_1(4)$ is chosen, and the random number generated is -5. Therefore, $x_2(4)$ = 3. It is infeasible because it conflicts with the initial status. Thus, if the number of ones is less than 4, we raise it up to 4. The new $x_2(4)$ = 4.

Then we check the feasibility. $x_2(2)$ and $x_2(3)$ exceed $x_2(4)$. We decrease the number to 4. The next feasible solution is

$$x_2 = [4 \quad 4 \quad 4 \quad 4 \quad 7 \quad 7 \quad 7 \quad 9 \quad \ldots \quad 28].$$

This random generator has one major advantage. Every generated neighborhood solution would have no conflict with the constraints and it is feasible. This repairing process is used to avoid infeasible solutions that violate the precedence constraints. Infeasible solutions will not be evaluated, so the processing time can be reduced significantly.

4-3 Temperature Parameter and Cooling Schedule

The acceptance probability considered in this problem is the one defined by Metropolis (1953):

$$P = \exp\left(-\frac{f(x) - f(x')}{T}\right) \quad \text{if } f(x) > f(x') \text{ and}$$

$$P = 1 \qquad \text{otherwise}$$

This form is used in many other papers that apply Simulated Annealing.

The initial temperature is computed by Triki's (2005) method, $T_0 = -\frac{\overline{\Delta f}^{(+)}}{\ln(1/2)}$, where $\overline{\Delta f}^{(+)}$ is the average change in cost over all bad moves. For the cooling schedule, typically the slower it is, the better result we get. In this particular problem, since running one iteration only takes approximately 0.1 seconds, we choose

0.99 to be our cooling rate. The temperature geometrically decreases every 5 iterations.

4-4 Stopping Criteria

Stopping criteria are also important issues in using Simulated Annealing. Some of the typical criteria include

- Maximum number of iterations reached.
- No change in the current solution for a very long time.
- The temperature is very low or the frozen state is reached, where no possibility of downhill move or no change in the objective function is observed.

The first two criteria are used in this problem. A small modification is made for the second one. We use the "best so far" solution instead of the current solution. The best so far (BSF) solution is stored, so that when ever an annealing process is stopped that configurations can be retrieved. It is possible that simulated annealing search might have moved from a global maximum during the initial stages of cooling and hence to avoid such a situation BSF solution is constantly stored in the memory.

4-5 SA Implementation Model

Step 1: randomly generate a feasible initial solution x_0 and calculate $f(x_0)$.

Step 2: from the current solution x_0, jump to its neighbor x' and calculate $f(x')$.

Step 3: compare $f(x_0)$ and $f(x')$.

If $f(x') > f(x_0)$, x' replaces x_0 to be the current solution.

Otherwise, randomly generate a number z between 0.01 and 0.99.

If $z < \exp(-\frac{f(x)-f(x')}{T})$, x' becomes the current solution.

Otherwise, do nothing.

Step 4: for every 5 iterations, reduce the temperature T by 1%, i.e. multiplying by 0.99.

Step 5: check termination rule.

Maximum iterations reached or stopping criteria reached.

If reached, algorithm stops. Otherwise, return to Step 2.

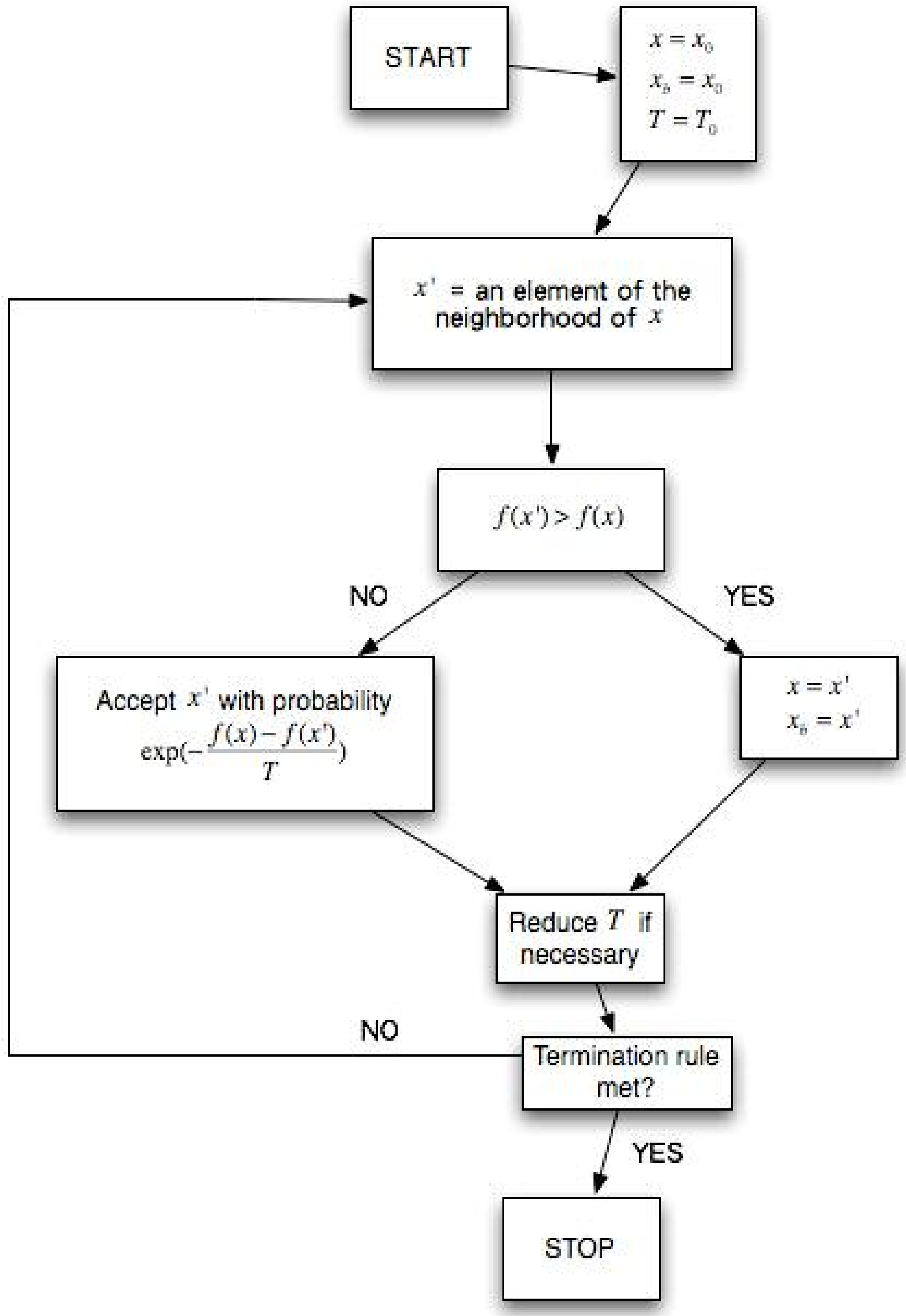

Fig. 4.1 SA Implementation Model

Chapter 5: Numerical Results

The procedure was coded with MATLAB 7.2.0 and run on an IBM Laptop with a 1.60 GHz Pentium R processor and 1.00 Gigabytes of RAM. Since running a 30-station route over a 30-year analysis period takes considerable time, a very large number of iterations is needed to converge while using Simulated Annealing. Several problem cases were tested: an unconstrained case, a revenue constrained-case and a revenue-and-budget-constrained case.

5-1 Description of Input Parameter Values

Table 5.1 Simulation Inputs

Variable	Description	Unit	Baseline
-	O/D matrix	-	
-	Demand model	-	
-	Matrix of demand growth rate	-	
-	Taxation ratio for operation	-	70%
-	Interest rate	-	5%
V	Cruise speed	miles/hr	40
d	Station spacing	Mile	
t_d	Dwell time	Hour	1/60
u_W	Unit cost of user waiting time	\$/passenger-hr	10
u_I	Unit cost of user in-vehicle time	\$/passenger-hr	5
u_T	Hourly operating cost	\$/vehicle-hr	300
n_C	Number of cars needed per train	cars/train	6
-	Operating hours per day	hrs/day	18
k	Capital cost for station and rail line	\$	
u_L	Maintenance unit cost	\$/passenger-mile	0.15

Optimization Input:

- Input decision (initial feasible solution)
- Initial temperature
- Cooling rate
- Threshold (maximum iterations)
- Stopping criteria

5-2 Unconstrained Case

Overall, the algorithm worked quite well. When running the SA about 20 times for exactly the same parameters and number of iterations, the results converged to the same solution more than 95% of the time. The solution is [4 27 27 … 27] and the objective value is $8.6839*10^9$. In average, running SA one time with 50k iterations threshold and 15k iterations stopping criterion takes approximately 4500 seconds.

Figure 5.1 illustrates the trace of the objective value changes. The X axis represents iterations and Y axis represents the net present worth of total net benefits. At the beginning of the annealing process, the temperature is high. The objective value fluctuates greatly. It escapes the starting relative maximum fairly easily to get trapped in a different local maximum. As temperature decreases with additional iterations, the oscillations decrease.

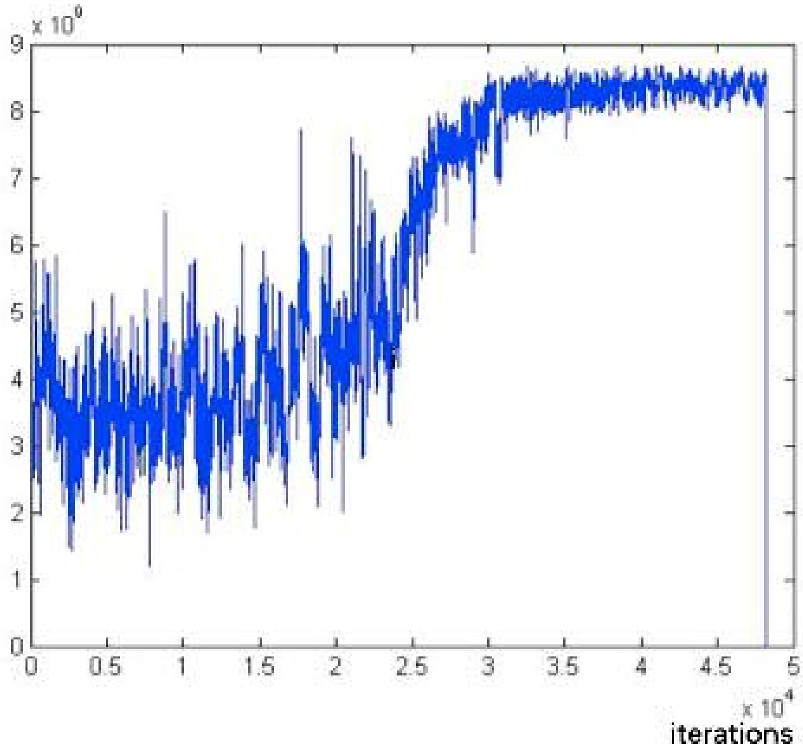

Fig. 5.1 Unconstrained Objective Value Fluctuations over Iterations

Figure 5.2 shows the optimized solution obtained for the unconstrained case. Surprisingly, there is only one phase which consists of adding 23 links in year 2. Since it is assumed that we have unlimited funds for extensions, this answer implies we should add links as soon as possible if the demand is enough. Demand at stations 28, 29 and 30 is too low, so the route stops at station 27.

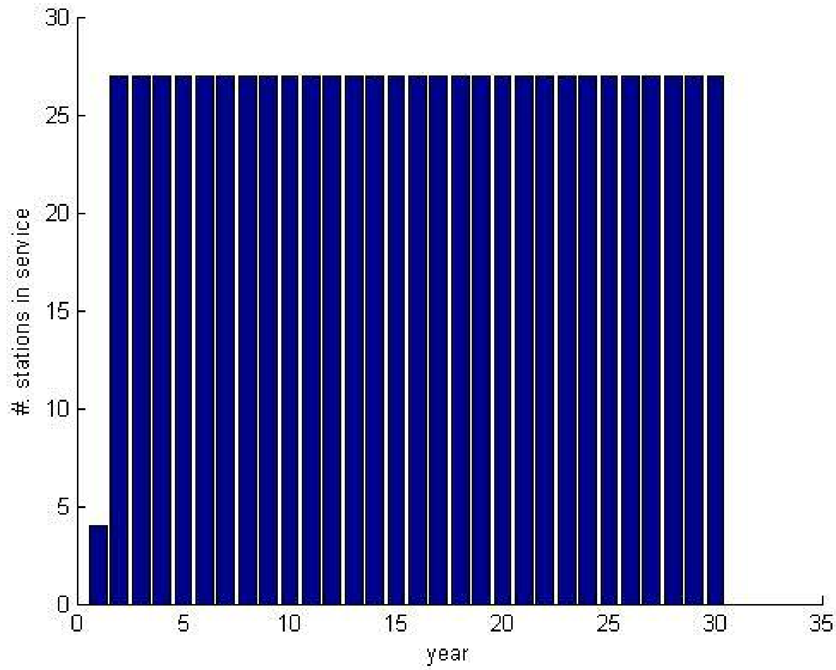

Fig. 5.2 Optimized Solution for Unconstrained Case

Figure 5.3 presents different measures of effectiveness. Figure 5.3 (a) shows the average ridership per day for two alternatives in each year. Comparing these two alternatives, the optimized one has a jump in year 2 and afterward increases much faster. The steep slope of the increase is due to higher demand growth rate as the transit route is extended further. Figures 5.3 (b) and (c) plot both supplier and user costs and the fraction of total costs. Both maintenance costs and in-vehicle costs increase significantly, since they are related to ridership. Maintenance and in-vehicle costs increase as ridership increases. Operating and user waiting cost are related to headway. They almost overlap, since the Y axis units are very large. It seems both costs have the same value. Figure 5.3 (d) shows the net present worth in each year and the optimized phases. The discounted net benefits respond to the addition of links. In year 2 the negative value is due to the construction costs. Figure 5.3 (e) illustrates the increase of passenger-miles. It has the same growth trend as the demand, with a jump in year 2 and significant rise afterwards.

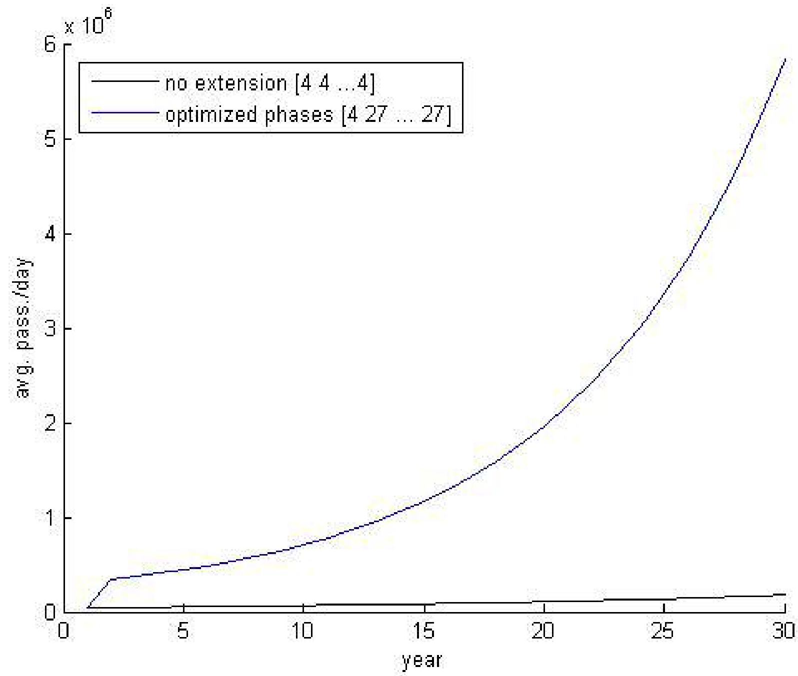

Fig. 5.3 (a) Average Passengers per Day

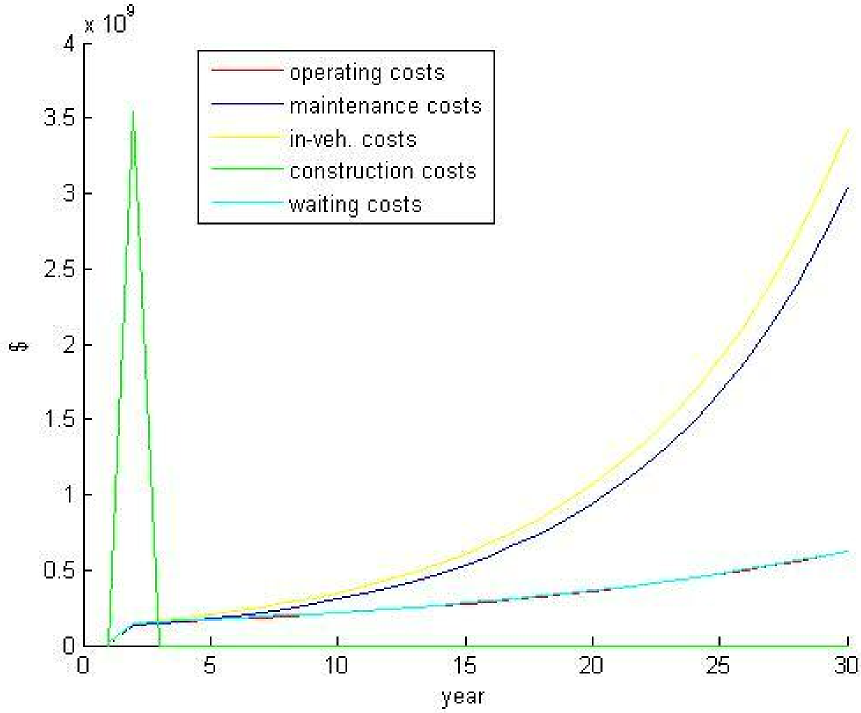

Fig. 5.3 (b) Supplier and User Costs

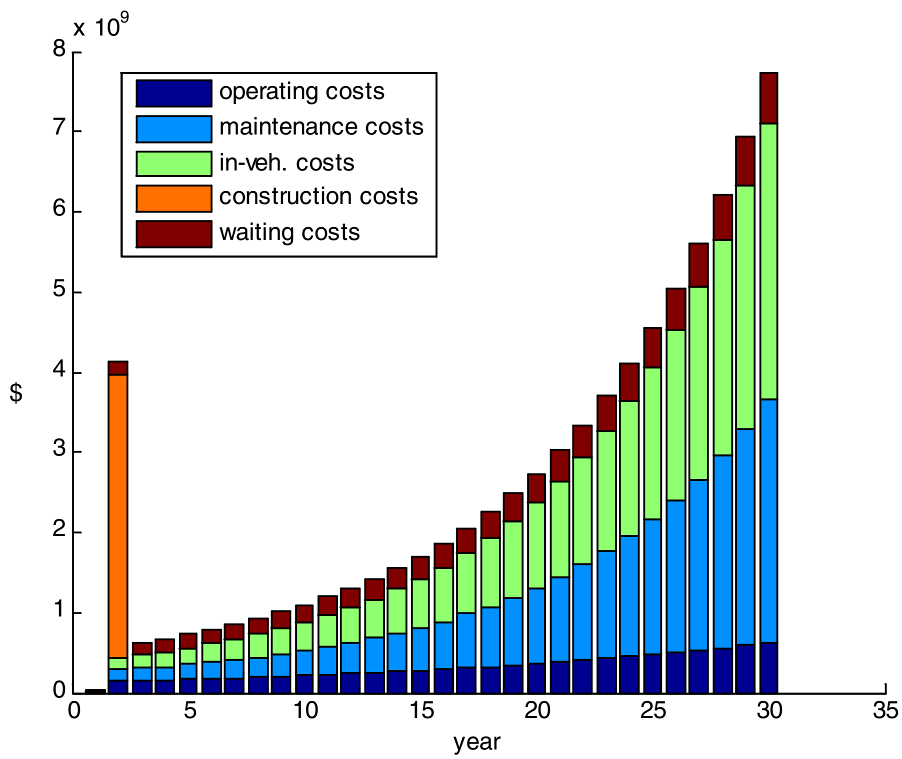

Fig. 5.3 (c) Breakdown of Costs for the Unconstrained Case

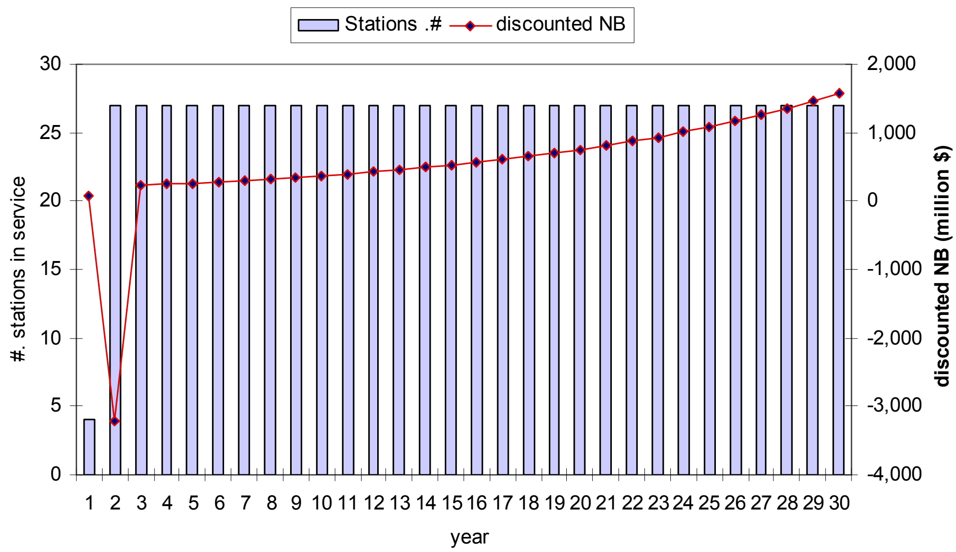

Fig. 5.3 (d) Discounted Net Benefits and Optimized Phases

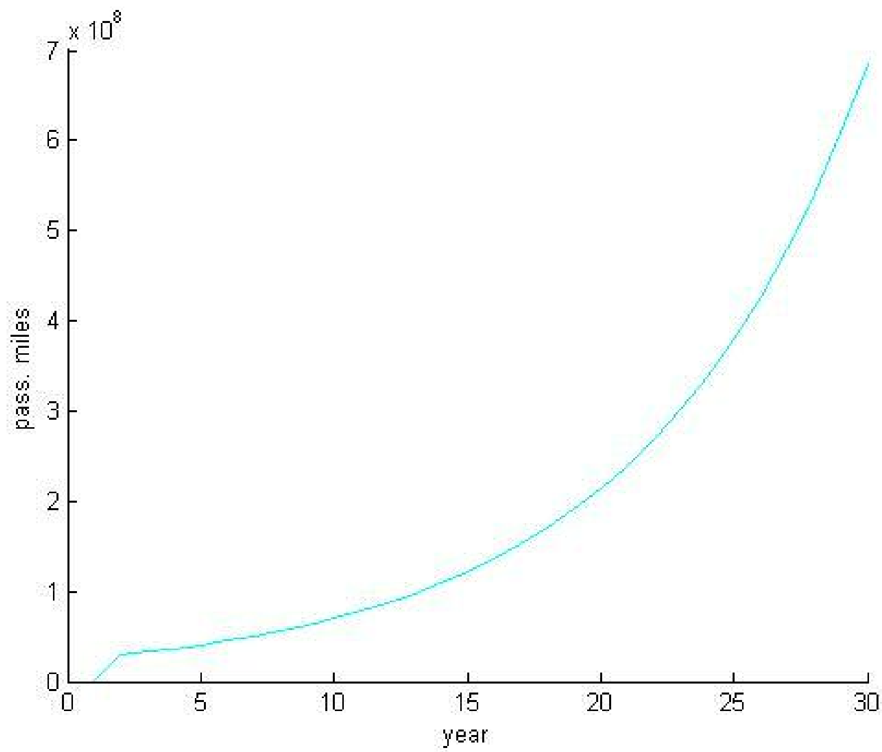

Fig. 5.3 (e) Passenger-miles in Years 0~30

Figure 5.4 compares different alternatives. In the upper one, the green line is the optimized solution found for the unconstrained case. The black line is the case without addition of links, which has only 4 stations in service for the 30 years horizon. The drop in year 2 indicates capital costs for extension. If the transit route is extended to link 27 in year 2, the net present worth will increase much faster than without an extension. From the upper graph, someone might argue for extending the transit route in year 17. The idea is that we always go for the alternative which has higher net benefits. However, this idea does not consider the capital investments. In the lower graph two more alternatives are added: Alternative 1 (red) extends to link 27 in year 17; alternative 2 (blue) extends to link 30 in year 2. None of them has higher objective value than the green line. Each line implies a different phase size and implementation time. That is why we cannot just choose the higher curve.

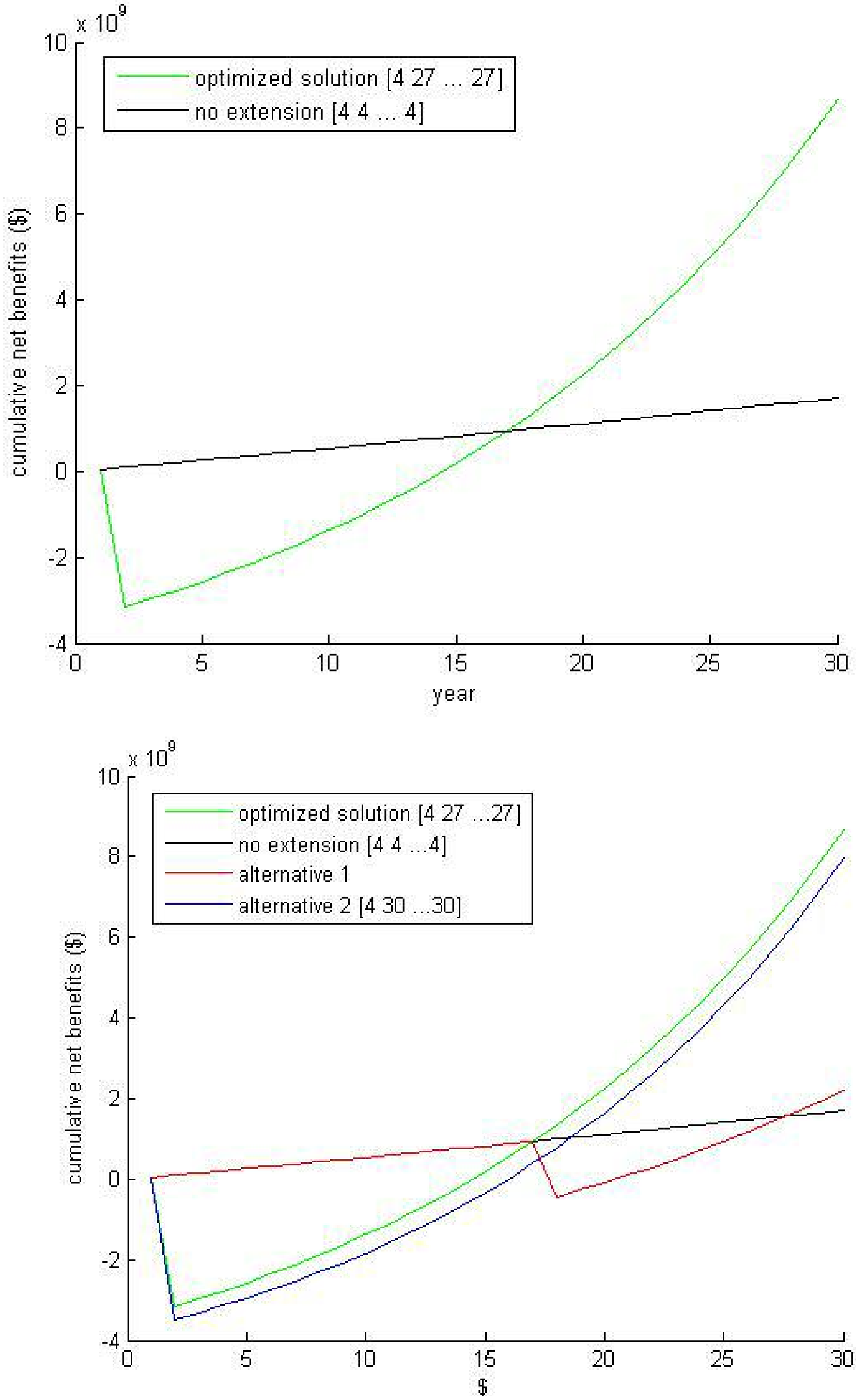

Fig. 5.4 Comparison of Alternatives for the Unconstrained Case

From Figure 5.4, without considering budget constraints, the solution would be adding links in year 2 as long as the demand is sufficient. Thus, if more is invested initially, much more will be earned later, although we might have a deficit in the early years.

In the next section the sensitivity to the demand and analysis period is analyzed. This preliminary run is intended to validate our answer. Detailed sensitivity analyses are provided in the next chapter.

5-2-1 Sensitivity to Demand

Table 5.2 shows the sensitivity analysis for different demand levels. The base level is 100% as shown in the table. When reducing the demand to 45% of the base level, the optimized solution just extends to link 15. When reducing the demand to 30% of the base level, the solution keeps only 4 stations for entire time frame. However demand changes, the nature of the solution, which schedules an extension in year 2, does not change.

Table 5.2 Effects of Demand

Demand	Cumulative Net Benefits	Optimized Solution
200%	2.32E+10	[4 27 … 27]
100%	3.27E+10	[4 27 … 27]
70%	4.49E+09	[4 27 … 27]
60%	3.13E+09	[4 27 … 27]
50%	1.79E+09	[4 27 … 27]
45%	1.39E+09	[4 15 … 15]
40%	9.90E+08	[4 15 … 15]
30%	4.25E+08	[4 4 … 4]

5-2-2 Sensitivity to Analysis Period

Table 5.3 shows the sensitivity solutions to different analysis periods. Longer analysis periods result in more links added in year 2. When extending the analysis horizon to 50 years, the optimized solution extends the transit route to link 29. When shortening the analysis horizon to 20 years, the optimized solution just extends the transit route to link 15. For 10 analysis years, the optimized solution has no extension for the entire time horizon. 30 years period is chosen in this thesis. If a longer analysis period is used, the life-cycle of rolling stock has to be considered in the model (e.g. replacement of old vehicles).

Table 5.3 Effects of Different Analysis Periods

Analysis Period (Year)	Cumulative Net Benefits	Optimized Solution
50	4.47E+10	[4 29 … 29]
40	2.06E+10	[4 27 … 27]
30	3.27E+10	[4 27 … 27]
25	5.00E+09	[4 27 … 27]
20	2.66E+09	[4 15 … 15]
15	1.32E+09	[4 15 … 15]
10	5.40E+08	[4 4 … 4]

5-2-3 Sensitivity to Growth Rate

Theoretically, if the demand is very low, the extension schedule should be delayed. The solution rule that schedules an extension in year 2 is due to higher growth rate toward the end of the transit route. This section examines the changes in solution when we set the equal demand growth rate before and after extension.

The optimized solution adds 11 links in year 2. From Figure 5.5, the optimized solution in the previous case, which extends to link 27 in year 2, is even worse than no extension. The optimized solution pattern obtained in the unconstrained case still does not change. The only difference is in the length of the extension. The reason will be explained in the following section.

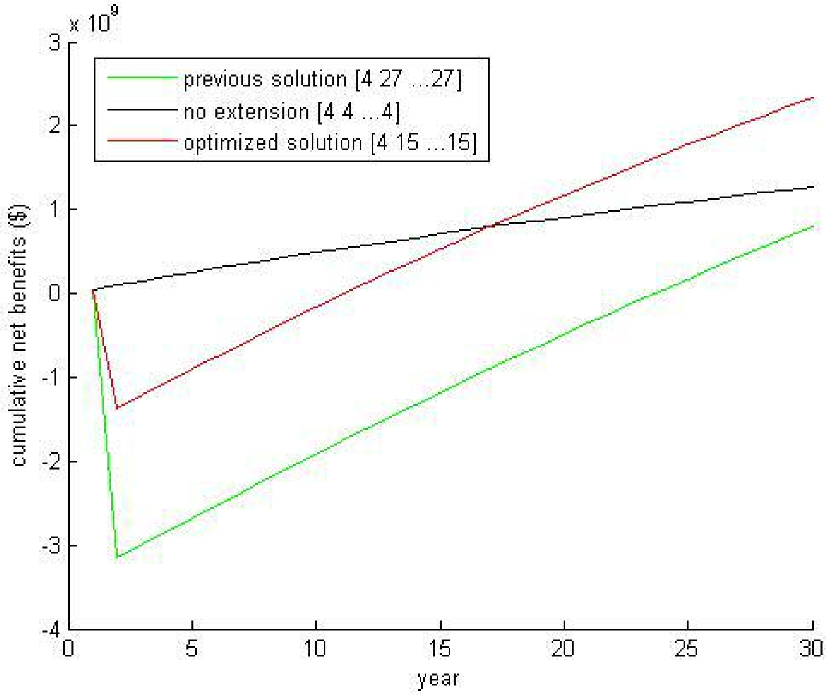

Fig. 5.5 Effects on Same Growth Rate before and after Extensions

Figure 5.6 shows the average ridership for different alternatives. The lower line represents an alternative with no extension, and the upper line represents the optimized alternative obtained in this case. Again, the optimized solution has a jump in year 2 and increases faster than the alternative with no extension. Even for the same growth rate after extension, the demand increases faster than when doing nothing. That occurs because some potential demand has been converted to actual demand.

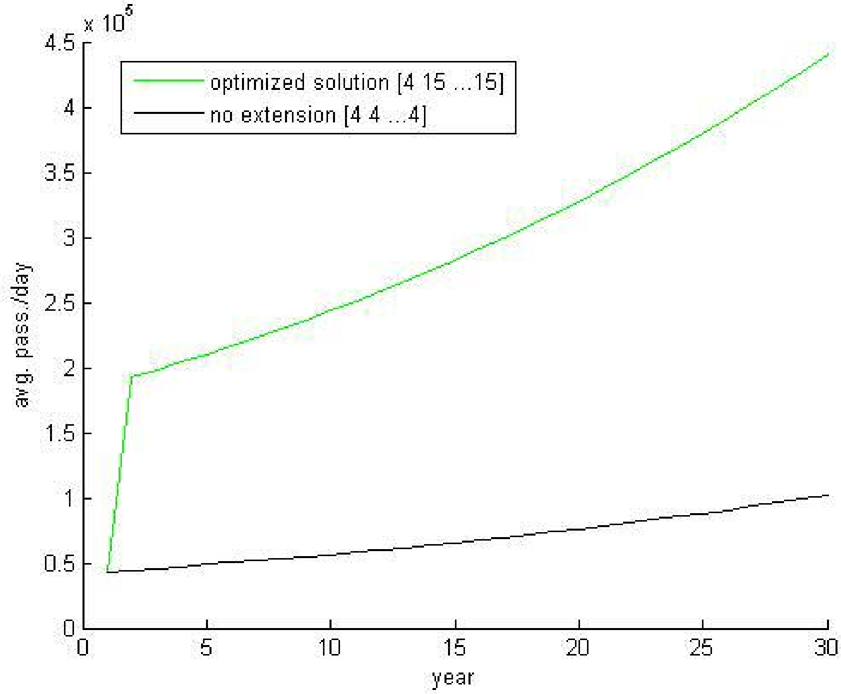

Fig. 5.6 Ridership for Different Alternatives

5-2-4 Marginal Analysis

A concept of marginal net benefits is presented to evaluate different alternatives. First, the marginal net benefits of adding link 5 is addressed. Three different alternatives are examined: (1) maintaining current state for the entire analysis period, i.e. no extension (4 stations in service); (2) adding link 5 in year 2 and (3) adding link 5 in year 5. Table 5.4 summarizes the results.

Compared with these three alternatives, Table 5.4 shows that the order of the net present worth (NPW) is alternative 2 > alternative 3 > alternative 1. That indicates alternative 2 is the preferable one. When adding link 5 in year 2, the marginal net benefits are 3.32E+08. When adding link 5 in year 5, the marginal net benefits decrease to 2.80E+08. Without considering the capital costs, adding link 5 always has positive net benefits in each year. Since capital costs account for large fraction of total costs, extending links which have enough demand as soon as possible can dilute the effects of sunk costs and achieve higher NPW. That is the case when potential demand is high enough. Next, a counterexample is discussed.

Table 5.4 Marginal Analysis of Adding Link 5

	Alternative 1			Alternative 2			Alternative 3		
Year	Total Benefits	Total Costs	Discounted NB	Total Benefits	Total Costs	Discounted NB	Total Benefits	Total Costs	Discounted NB
1	7.80E+07	3.11E+07	4.67E+07	7.80E+07	3.13E+07	4.67E+07	7.80E+07	3.13E+07	4.67E+07
2	8.19E+07	3.23E+07	4.72E+07	1.11E+08	2.39E+08	-1.22E+08	8.19E+07	3.23E+07	4.72E+07
3	8.60E+07	3.34E+07	4.77E+07	1.16E+08	4.57E+07	6.39E+07	8.60E+07	3.34E+07	4.77E+07
4	9.03E+07	3.45E+07	4.82E+07	1.22E+08	4.72E+07	6.46E+07	9.03E+07	3.45E+07	4.82E+07
5	9.48E+07	3.56E+07	4.87E+07	1.28E+08	4.88E+07	6.52E+07	1.26E+08	2.43E+08	-9.63E+07
6	9.96E+07	3.67E+07	4.92E+07	1.34E+08	5.04E+07	6.59E+07	1.32E+08	4.98E+07	6.48E+07
7	1.05E+08	3.80E+07	4.97E+07	1.41E+08	5.21E+07	6.65E+07	1.39E+08	5.15E+07	6.54E+07
8	1.10E+08	3.92E+07	5.02E+07	1.48E+08	5.38E+07	6.71E+07	1.46E+08	5.32E+07	6.60E+07
9	1.15E+08	4.05E+07	5.06E+07	1.56E+08	5.56E+07	6.77E+07	1.53E+08	5.50E+07	6.66E+07
10	1.21E+08	4.18E+07	5.10E+07	1.63E+08	5.75E+07	6.83E+07	1.61E+08	5.69E+07	6.71E+07
11	1.27E+08	4.32E+07	5.15E+07	1.72E+08	5.95E+07	6.88E+07	1.69E+08	5.88E+07	6.77E+07
12	1.33E+08	4.47E+07	5.19E+07	1.80E+08	6.11E+07	6.94E+07	1.78E+08	6.08E+07	6.83E+07
13	1.40E+08	4.62E+07	5.23E+07	1.89E+08	6.36E+07	6.99E+07	1.86E+08	6.29E+07	6.88E+07
14	1.47E+08	4.78E+07	5.27E+07	1.99E+08	6.58E+07	7.04E+07	1.96E+08	6.51E+07	6.93E+07
15	1.54E+08	4.94E+07	5.31E+07	2.09E+08	6.81E+07	7.09E+07	2.06E+08	6.73E+07	6.98E+07
16	1.62E+08	5.11E+07	5.35E+07	2.19E+08	7.05E+07	7.14E+07	2.16E+08	6.97E+07	7.03E+07
17	1.70E+08	5.28E+07	5.38E+07	2.30E+08	7.30E+07	7.19E+07	2.27E+08	7.21E+07	7.08E+07
18	1.79E+08	5.46E+07	5.42E+07	2.41E+08	7.55E+07	7.24E+07	2.38E+08	7.46E+07	7.13E+07
19	1.88E+08	5.65E+07	5.45E+07	2.54E+08	7.82E+07	7.29E+07	2.50E+08	7.72E+07	7.17E+07
20	1.97E+08	5.85E+07	5.49E+07	2.66E+08	8.09E+07	7.33E+07	2.62E+08	8.00E+07	7.22E+07
21	2.07E+08	6.05E+07	5.52E+07	2.79E+08	8.38E+07	7.37E+07	2.75E+08	8.28E+07	7.26E+07
22	2.17E+08	6.26E+07	5.56E+07	2.93E+08	8.68E+07	7.42E+07	2.89E+08	8.58E+07	7.30E+07
23	2.28E+08	6.48E+07	5.59E+07	3.08E+08	8.99E+07	7.46E+07	3.04E+08	8.88E+07	7.34E+07
24	2.40E+08	6.71E+07	5.62E+07	3.24E+08	9.32E+07	7.50E+07	3.19E+08	9.20E+07	7.38E+07
25	2.52E+08	6.95E+07	5.65E+07	3.40E+08	9.65E+07	7.54E+07	3.35E+08	9.53E+07	7.42E+07
26	2.64E+08	7.19E+07	5.68E+07	3.57E+08	1.00E+08	7.58E+07	3.52E+08	9.88E+07	7.46E+07
27	2.77E+08	7.45E+07	5.71E+07	3.75E+08	1.04E+08	7.62E+07	3.69E+08	1.02E+08	7.50E+07
28	2.91E+08	7.72E+07	5.74E+07	3.93E+08	1.07E+08	7.65E+07	3.88E+08	1.06E+08	7.54E+07
29	3.06E+08	7.99E+07	5.76E+07	4.13E+08	1.11E+08	7.69E+07	4.07E+08	1.10E+08	7.57E+07
30	3.21E+08	8.28E+07	5.79E+07	4.34E+08	1.16E+08	7.73E+07	4.27E+08	1.14E+08	7.61E+07
Total	5.18E+09	1.58E+09	1.59E+09	6.97E+09	2.37E+09	1.92E+09	6.79E+09	2.31E+09	1.87E+09

For the counter example, two alternatives are considered here: (1) the optimized solution in the unconstrained case, i.e. extending the transit route to link 27 in year 2; (2) extending the route to link 27 in year 2, and adding one more link in year 3. Table 5.5 summarizes the results.

Table 5.5 Marginal Analysis of Adding Link 28

Year	[4 27 27 … 27]			[4 27 28 … 28]		
	Total Benefits	Total Costs	Discounted NB	Total Benefits	Total Costs	Discounted NB
1	7.80E+07	3.13E+07	4.67E+07	7.80E+07	3.13E+07	4.67E+07
2	6.09E+08	4.02E+09	-3.25E+09	6.09E+08	4.02E+09	-3.25E+09
3	6.67E+08	5.22E+08	1.31E+08	6.84E+08	6.83E+08	6.00E+05
4	7.31E+08	5.66E+08	1.42E+08	7.50E+08	5.96E+08	1.34E+08
5	8.01E+08	6.15E+08	1.53E+08	8.23E+08	6.48E+08	1.44E+08
6	8.79E+08	6.69E+08	1.65E+08	9.05E+08	7.06E+08	1.56E+08
7	9.66E+08	7.29E+08	1.77E+08	9.95E+08	7.70E+08	1.68E+08
8	1.06E+09	7.95E+08	1.89E+08	1.10E+09	8.42E+08	1.80E+08
9	1.17E+09	8.69E+08	2.03E+08	1.21E+09	9.21E+08	1.94E+08
10	1.29E+09	9.50E+08	2.17E+08	1.33E+09	1.01E+09	2.07E+08
11	1.42E+09	1.04E+09	2.32E+08	1.47E+09	1.11E+09	2.22E+08
12	1.56E+09	1.14E+09	2.47E+08	1.62E+09	1.22E+09	2.38E+08
13	1.73E+09	1.25E+09	2.64E+08	1.80E+09	1.34E+09	2.54E+08
14	1.91E+09	1.38E+09	2.81E+08	1.99E+09	1.48E+09	2.71E+08
15	2.11E+09	1.52E+09	3.00E+08	2.20E+09	1.63E+09	2.90E+08
16	2.34E+09	1.67E+09	3.20E+08	2.44E+09	1.80E+09	3.09E+08
17	2.59E+09	1.85E+09	3.40E+08	2.71E+09	1.99E+09	3.30E+08
18	2.87E+09	2.04E+09	3.63E+08	3.01E+09	2.20E+09	3.52E+08
19	3.19E+09	2.26E+09	3.86E+08	3.34E+09	2.44E+09	3.75E+08
20	3.54E+09	2.50E+09	4.11E+08	3.72E+09	2.71E+09	4.00E+08
21	3.93E+09	2.77E+09	4.38E+08	4.14E+09	3.01E+09	4.26E+08
22	4.37E+09	3.07E+09	4.67E+08	4.61E+09	3.35E+09	4.54E+08
23	4.87E+09	3.41E+09	4.97E+08	5.14E+09	3.72E+09	4.84E+08
24	5.42E+09	3.79E+09	5.29E+08	5.74E+09	4.15E+09	5.16E+08
25	6.04E+09	4.22E+09	5.64E+08	6.41E+09	4.63E+09	5.51E+08
26	6.74E+09	4.70E+09	6.01E+08	7.16E+09	5.17E+09	5.88E+08
27	7.52E+09	5.25E+09	6.41E+08	8.01E+09	5.78E+09	6.27E+08
28	8.40E+09	5.85E+09	6.83E+08	8.96E+09	6.46E+09	6.69E+08
29	9.39E+09	6.54E+09	7.29E+08	1.00E+10	7.23E+09	7.14E+08
30	1.05E+10	7.31E+09	7.77E+08	1.12E+10	8.10E+09	7.62E+08
Total	9.87E+10	7.33E+10	7.24E+09	1.04E+11	7.97E+10	6.81E+09

The marginal net benefits of adding link 28 in year 3 are 6.81E+09 – 7.24E+09 = -4.33E+08. The negative marginal net benefits indicate that alternative 2 is not acceptable. Since the economic benefit of link 28 is insufficient to add it, adding link 28 causes more negative impacts (e.g. user costs) than benefits (i.e. user benefits). Some links are not economically beneficial over a 30 years analysis period, but they may become over a longer. links 28 and 29 are added in the 50 years case, as shown in Table 5.3.

Marginal analysis is a key to economic analysis. It helps the decision makers look at the effects of a small change in the control variable. However, analyzing this problem by checking the marginal net benefits is too complicated. The marginal net benefits of adding one link are affected by the year in which the link is added, the current ridership, the potential demand of the link to be added and the growth rate of the link. However, it is still helpful to identify the solution pattern by the marginal analysis.

From different sensitivity tests and marginal analysis, the solution pattern that schedules an extension at the beginning corresponds to the result of Kolisch and Padman (2001). Delaying the extensions when demand is low might happen in a problem whose objective is profit maximization or cost minimization. If we always have enough money to add links and no other constraint, adding links with positive net benefits as soon as possible would maximize the net present worth (NPW).

5-3 Revenue-Constrained Case

In transit operation, fare revenue may be used in at least two ways. Some fraction of it may be used for covering operation costs; the other may be devoted to funding capital investments. In a real operation, if the revenue cannot balance the expense due to low demand, the transit operator tends to postpone the extension. Figure 5.7 shows the revenue used for operation and the supplier operating expense each year for the unconstrained case. The supplier expense consists of operating and maintenance costs. The supplier costs exceed 70% of revenue for the entire analysis period. Therefore, two kinds of constraints are tested here: 1) a revenue constraint; 2) a budget constraint. First, the revenue constraint is incorporated into the model. The budget constraint is added in the next section.

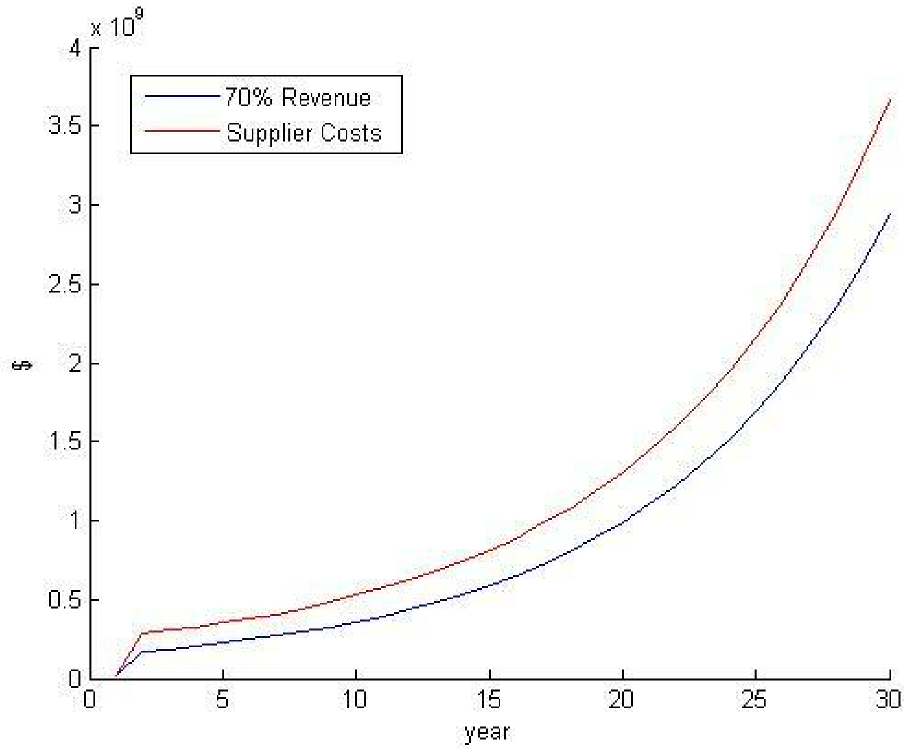

Fig. 5.7 Operating Expenses and Subsidies

For the constrained case, the problem becomes more complicate, so the threshold and stopping criterion iterations are increased to 100k and 40k. The objective value is $4.1242*10^9$. Running SA one time for the revenue-constrained case averagely takes 7600 seconds. Figure 5.8 shows the optimized solution for the revenue-constrained case. The optimized solution for the revenue-constrained case has 5 phases: phase I adds 4 links in year 2; phase II adds 1 link in year 4; phase III adds 1 link in year 7; phase IV adds 1 link in year 10, and the final phase adds 4 links in year 17. When considering the revenue constraints, the route only extends to link 15, and we have lower NPW compared with the unconstrained case, since extensions are postponed.

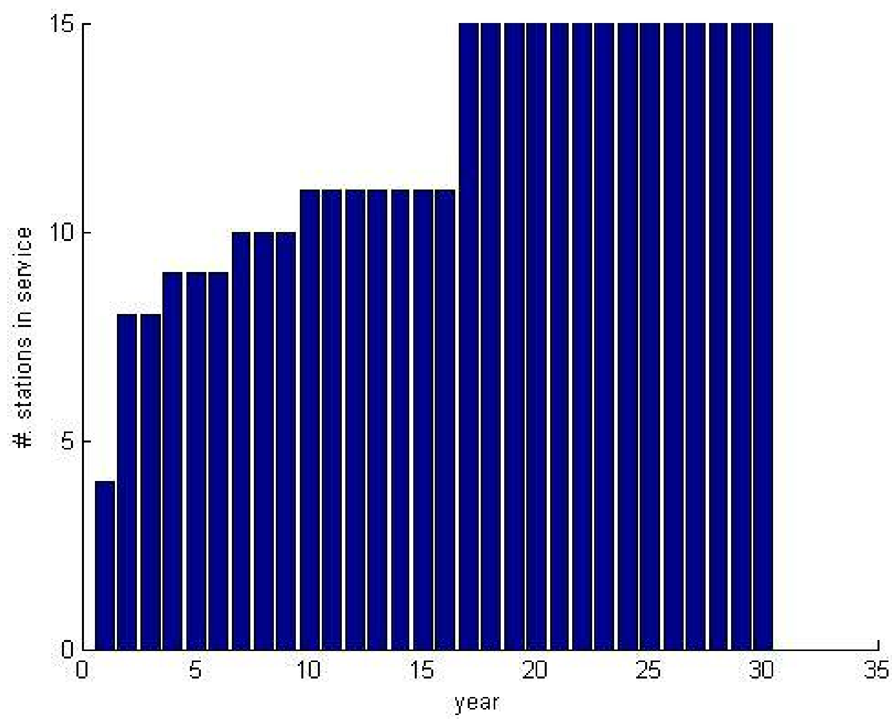

Fig. 5.8 Optimized Solution for Revenue-Constrained Case

Figure 5.9 shows that in this analysis operating expenses are lower than the revenue funds after incorporating revenue constraints.

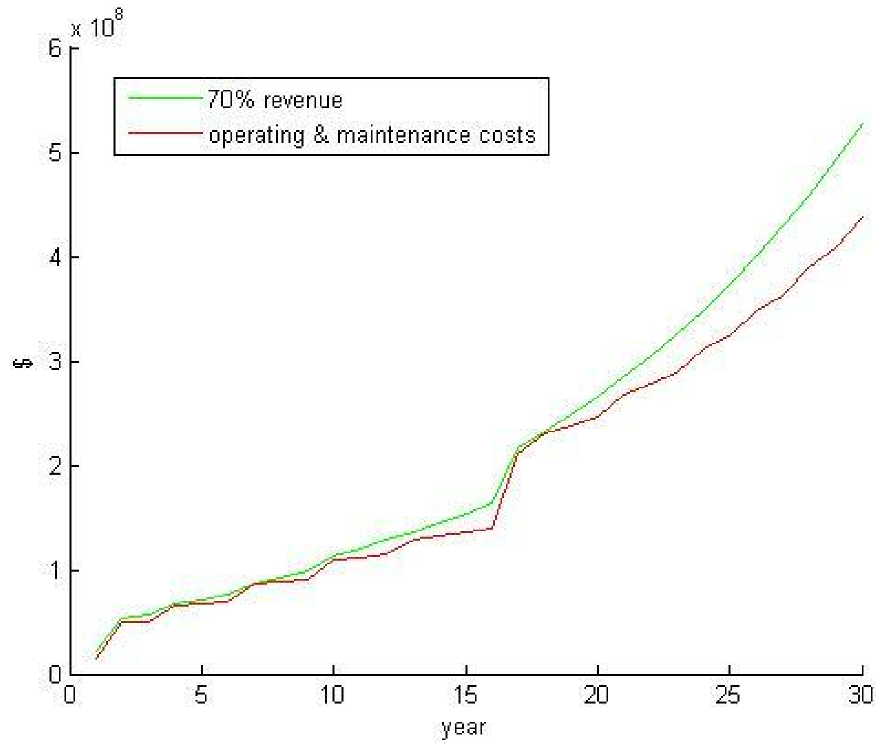

Fig. 5.9 Operating Expenses and Funds (Revenue-Constrained Case)

Figure 5.10 shows the discounted net benefits in each year and the optimized phases. The discounted net benefits respond to the addition of link. Each drop in discounted net benefits is due to addition of links.

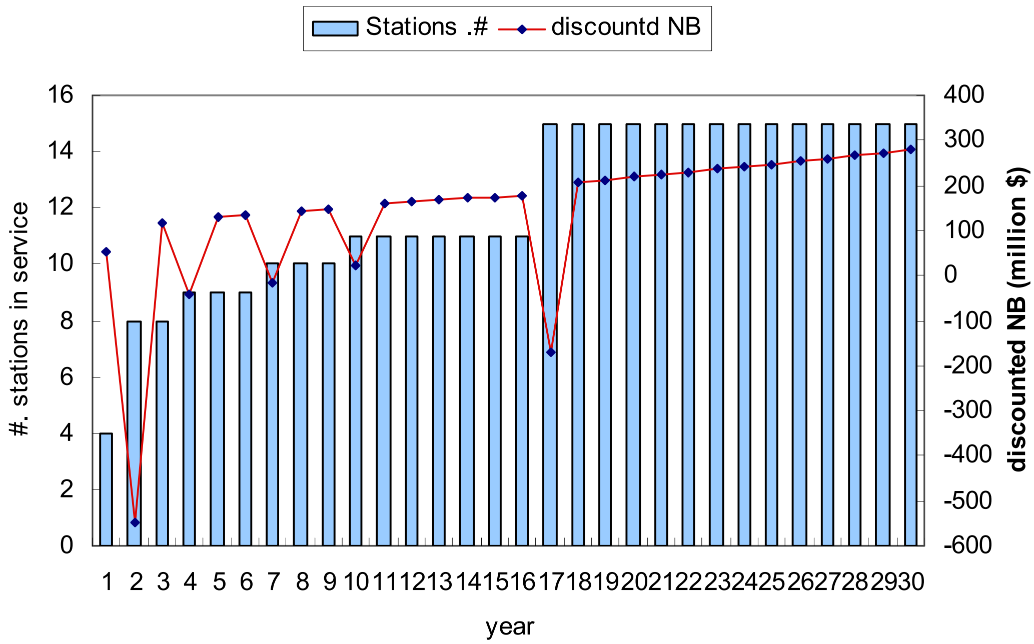

Fig. 5.10 Discounted Net Benefits and Optimized Phases

Figure 5.11 compares the optimized solution with the alternative which has no extension. In the first 8 years, the cumulative net benefits are negative. After year 8, the cumulative net benefits become positive. With more stations in service, the cumulative net benefits would increase at a higher rate.

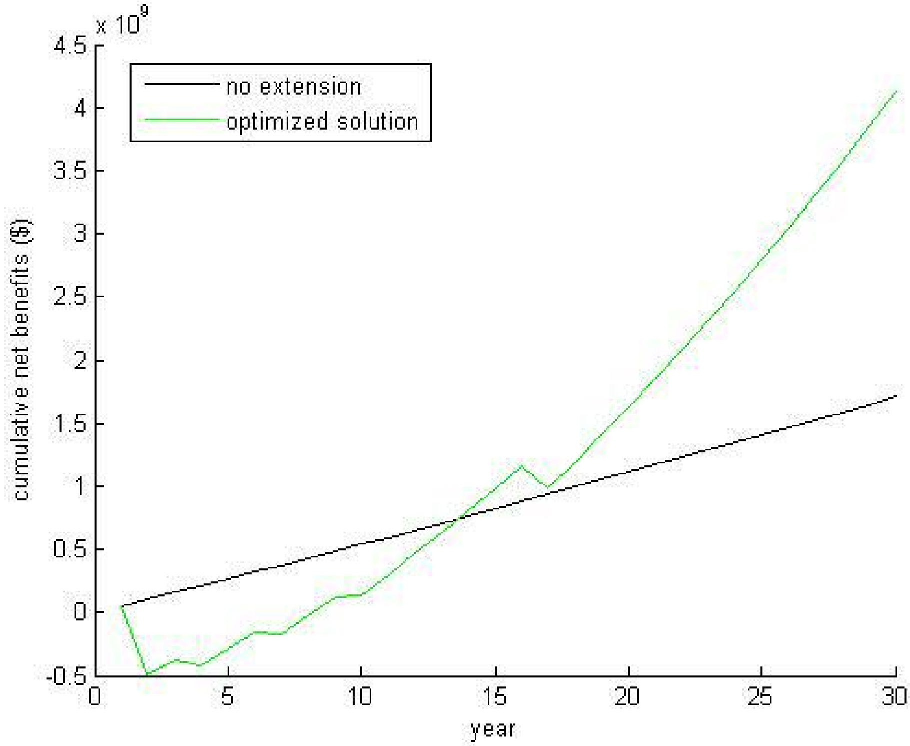

Fig. 5.11 Comparison of Alternatives for the Revenue-Constrained Case

5-4 Revenue-Budget-Constrained Case

In this section a budget constraint is added in the model. It is assumed that subsidies for capital investments are available in the beginning of each year. In addition, 70% of the fare collection is used for covering operational expenditure, and the rest of the fare collection is used for capital investments. Penalty methods are used for dealing with constraints. A 5% offset is added into both revenue and budget constraints. Adding such an offset is reasonable because we do not want to delay the construction just because of small shortfalls.

For the revenue-budget-constrained case, stopping criterion is increased to 100k iterations and the objective value is $4.0591*10^9$. Figure 5.12 traces of the objective value changes for the revenue-budget-constrained case. The trend in the constrained case is almost the same as in the previous case. *NPW* fluctuates dramatically at first, and oscillations decrease over later iterations. Negative objective values are due to the penalty method used in Simulated Annealing to deal with constraints. 9364 seconds are needed to get the optimized solution on an IBM Laptop with a 1.60 GHz Pentium R processor and 1.00 Gigabytes of RAM.

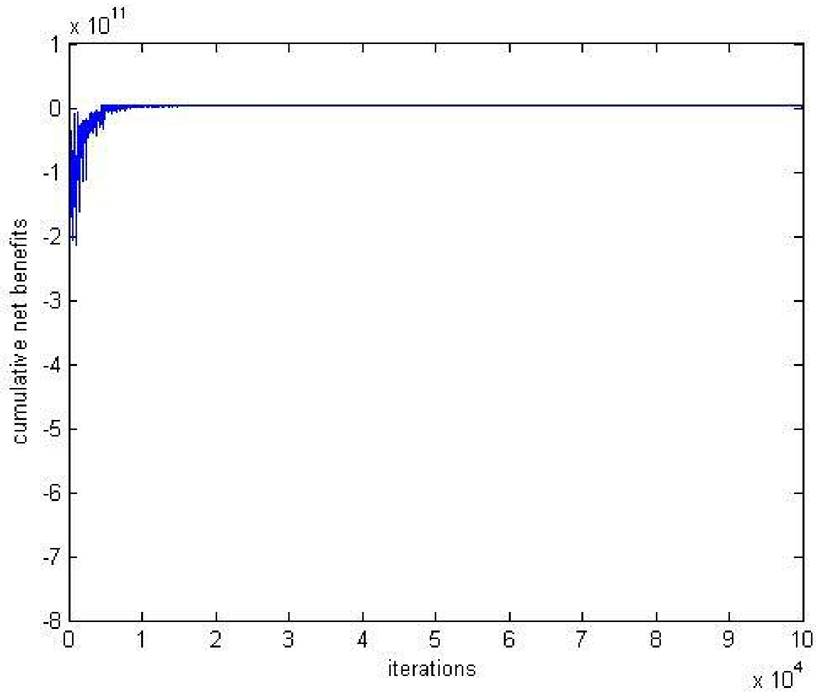

Fig. 5.12 Objective Value Fluctuations Constrained by Budget and Revenue over Iterations

Figure 5.13 shows the optimized phases for the case constrained by revenue and budget. The solution is slightly different than in the last case. There are six phases for this case: phase I adds 3 links in year 3; phase II adds 2 links in year 5; phase III adds 1 link in year 6; phase IV adds 1 link in year 9; phase V adds 3 links in year 13 and the last phase adds 1 link in year 14.

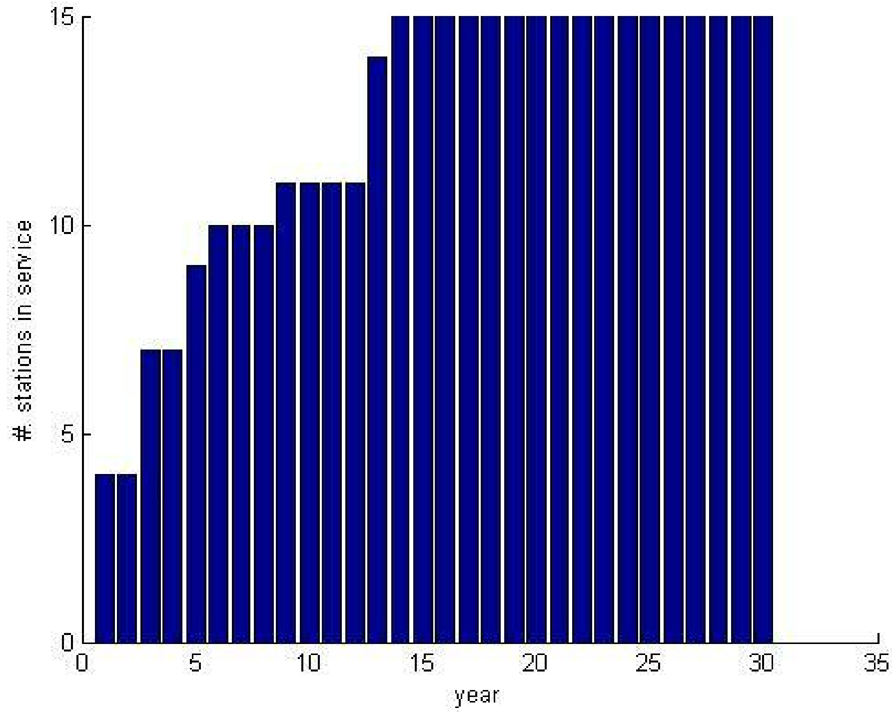

Fig. 5.13 Optimized Solution for the Case Constrained by Budget and Revenue

Table 5.6 summarizes the results. Additional links will increase not only the ridership, but also operation and maintenance costs. On the other hand, increased ridership due to the convenience of an additional link may generate substantial revenues. In addition, decreasing headway indicates that the service quality is improving through route extensions.

Table 5.6 Summary of the optimized solution for the revenue-budget-constrained case

Year		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
#. Stations in service		4	4	7	7	9	10	10	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
UB(10^8 \$)		0.8	0.8	1.7	1.8	2.5	2.9	3.0	3.2	3.7	4.0	4.2	4.5	5.6	6.5	7.0	7.5	8.0	8.5	9.1	9.8	10.5	11.2	12.0	12.9	13.8	14.8	15.9	17.1	18.3	19.6
TC	Cor (10^7 \$)	1.2	1.2	3.5	3.5	4.7	5.8	5.8	5.8	7.0	7.0	7.0	7.0	9.3	10.5	1.2	1.2	1.2	12.8	12.8	12.8	14.0	14.0	15.2	15.2	16.3	16.3	17.5	17.5	18.7	18.7
	Cm (10^7 \$)	0.4	0.4	1.2	1.3	2.0	2.6	2.7	2.9	3.6	3.9	4.1	4.4	6.8	8.1	8.8	9.5	10.2	11.0	11.8	12.7	13.7	14.8	16.0	17.2	18.6	20.0	21.6	23.3	25.2	27.2
	Cw (10^7 \$)	1.6	1.7	3.4	3.5	4.8	5.5	5.7	5.9	6.7	6.9	7.1	7.3	9.8	10.9	11.2	11.6	12.0	12.4	12.9	13.3	13.8	14.3	14.8	15.3	15.8	16.4	17.0	17.6	18.2	18.9
	Ci (10^7 \$)	0.5	0.6	1.6	1.7	2.6	3.3	3.5	3.8	4.7	4.9	5.3	5.6	8.4	10.0	10.8	11.6	12.5	13.5	14.5	15.6	16.8	18.1	19.5	21.0	22.7	24.5	26.4	28.4	30.7	33.1
	Ck (10^8 \$)			5.5		3.6	2.1			2.1				6.9	1.7																
Discounted NB (10^7 \$)		5.2	5.3	-40.2	9.8	-17.3	-3.0	13.7	14.0	0.9	15.4	15.7	16.1	-20.9	10.4	19.6	20.1	20.6	21.2	21.7	22.2	22.8	23.4	24.0	24.6	25.2	25.8	26.4	27.1	27.8	28.5
Headway (hr)		0.21	0.21	0.20	0.20	0.19	0.19	0.19	0.18	0.18	0.17	0.17	0.16	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.12	0.12	0.11	0.11	0.11	0.10	0.10	0.10
Fleet Size		1	1	3	3	4	5	5	5	6	6	6	6	8	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16
Avg. Pass./day (10^5 \$)		0.4	0.5	0.9	1.0	1.4	1.6	1.7	1.8	2.1	2.2	2.3	2.5	3.1	3.6	3.9	4.1	4.4	4.7	5.1	5.4	5.8	6.2	6.7	7.2	7.7	8.2	8.8	9.5	10.2	10.9
Revenue	Operation (10^7 \$)	0.9	1.0	2.0	2.1	3.0	3.4	3.6	3.9	4.5	4.8	5.1	5.4	6.8	7.8	8.4	8.9	9.6	10.2	11.0	11.7	12.6	13.5	14.4	15.5	16.6	17.8	19.1	20.5	22.0	23.6
	Investment (10^7 \$)	0.2	0.2	0.5	0.5	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.6	1.8	1.9	2.1	2.2	2.4	2.6	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.5	4.8	5.1	5.5
Deficit	Operation (10^6 \$)						3.83	0.62		1.77				3.13	4.22	9.43	2.55														
	Construction (10^6 \$)			1.21																											

Figure 5.14 presents different measures of effectiveness for the case constrained by budget and revenue. Figure 5.14 (a) shows the average ridership per day in each year for different alternatives. When links are added, demand increases at a faster rate since some potential demand is converted into actual demand. Figures 5.14 (b) and (c) plot both supplier and user costs and the breakdown of total costs. Construction costs account for most of total costs when links are added. Maintenance and user in-vehicle costs have similar growth trends, since they are estimated based on ridership. Operating and user waiting costs have similar growth trends, since they are evaluated based on headway. Maintenance costs are lower than operating costs for the first 20 years, and exceed operating costs in year 21. On average, the ratio of operating costs to maintenance costs is 1.03:1. Figure 5.14 (d) shows the discounted net benefits in each year and the optimized phases. The discounted net benefits drop significantly when links are added but bounce back with a higher value the following year.

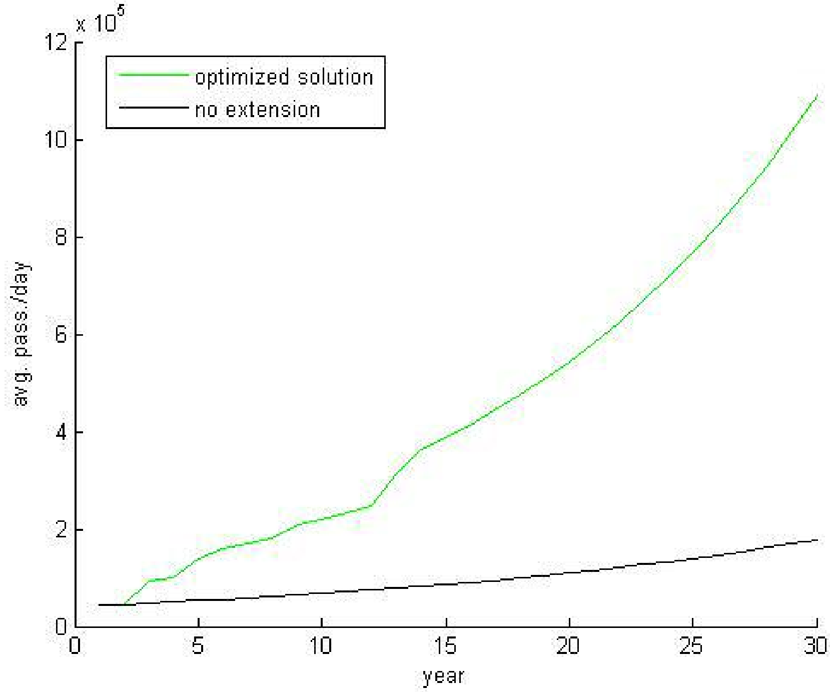

Fig. 5.14 (a) Average Passengers per Day

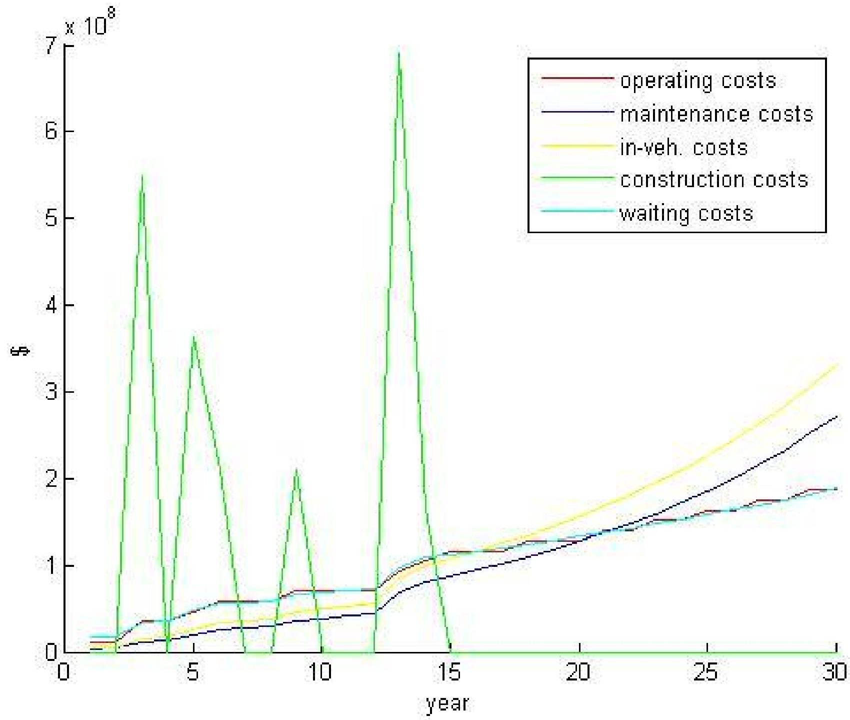

Fig. 5.14 (b) Supplier and User Costs

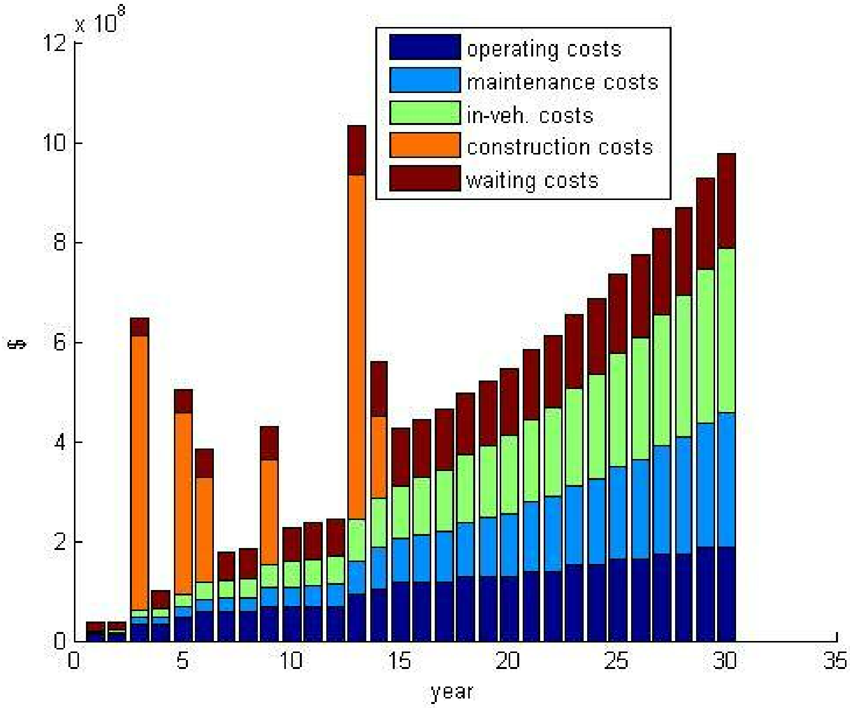

Fig. 5.14 (c) Breakdown of Costs for the Case Constrained by Revenue and Budget

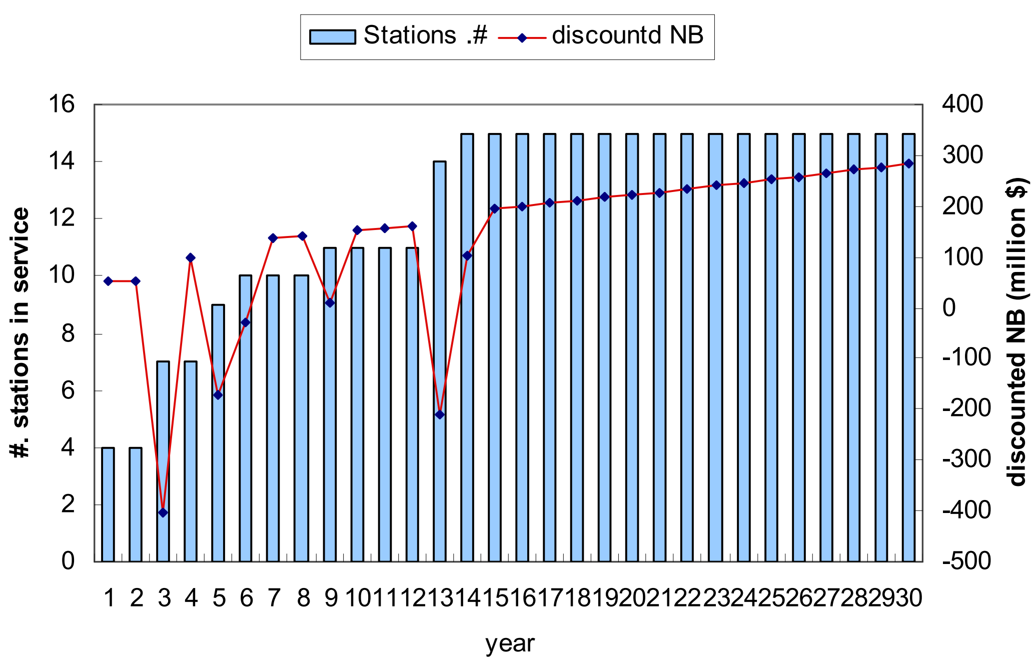

Fig. 5.14 (d) Discounted Net Benefits and Optimized Phases

Figure 5.15 shows the effects after 5% offset is applied. The circled parts show in what years the expenses exceed the operational funds. Table 5.6 presents the data on deficit spending. This indicates in what year the transit operator will be short of funds and the amount of money that would have to be borrowed.

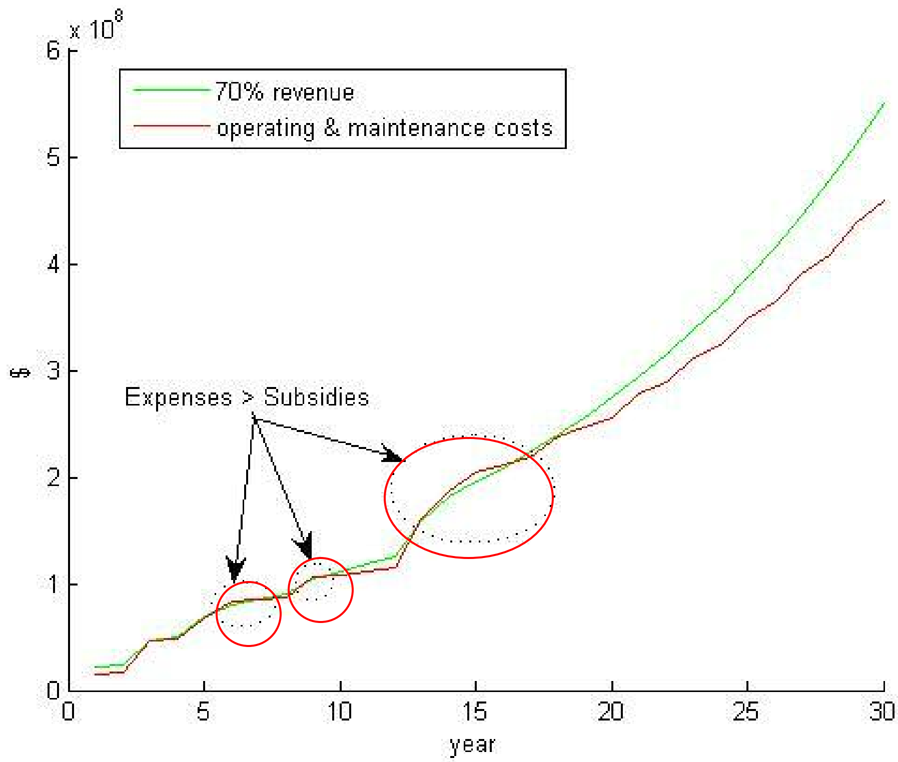

Fig. 5.15 Revenue Constraint offset

Figure 5.16 shows the available budgets for capital investments in each year. In year 3, the budget is about 1.2 million dollars short for implementing Phase I which adds 3 links. By adding 5% offset into revenue and budget constraints, a more flexible development plan can achieve higher net benefits. Through Figures 5.15, 5.16

and Table 5.6, it is clearly shown when funds will be insufficient if the extension plan is applied.

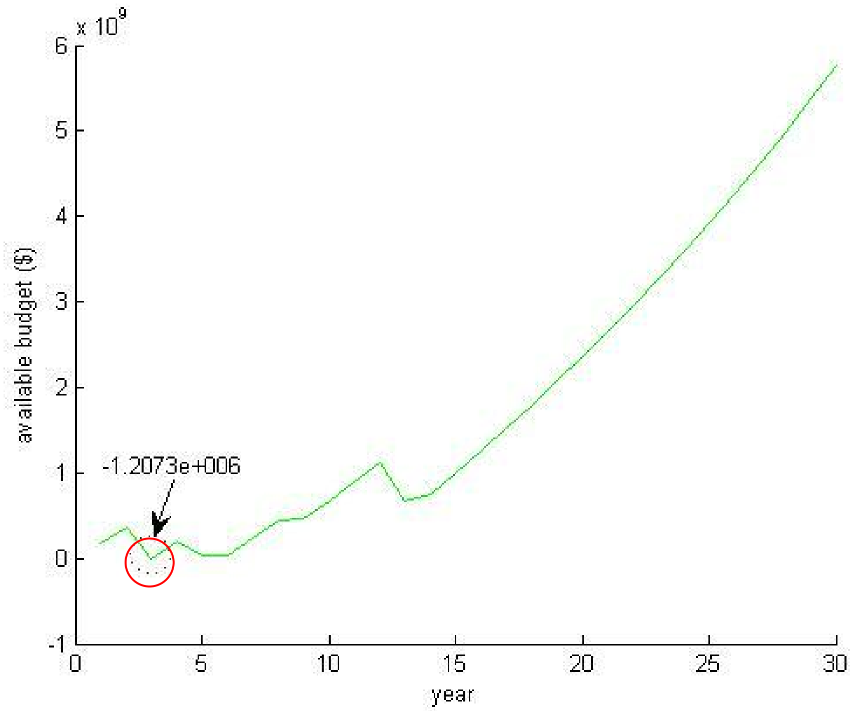

Fig. 5.16 Budget Constraint Offset

Figure 5.17 shows the net present worth for all three cases. In it as more constraints are applied net benefits decrease, as expected. However, the differences in NPW between revenue-constrained case and case constrained by revenue and budget are small. There are probably two reasons: first, the 5% offset brings the answers in two cases fairly close; second, the revenue constraint dominates in the numerical example. This is shown in Figures 5.15 and 5.16. In Figure 5.16, the available budget for new investments is quite sufficient after year 15. The operational expenses and funds are fairly close over 30 years, as shown in Figure 5.15. Adding a budget constraint does not bind the solution. Compared with the unconstrained and revenue-budget-constrained cases, the NPW in the case constrained by revenue and budget is nearly one third of that in the unconstrained case. NPW is significantly affected by the construction phases.

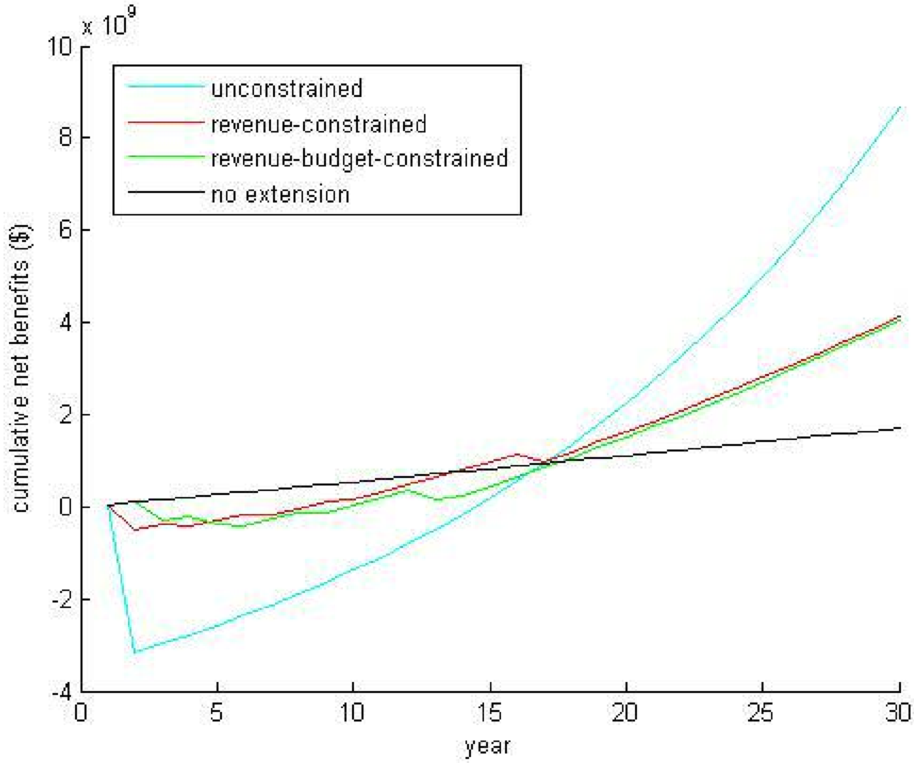

Fig. 5.17 Comparison of All Cases

5-5 Discussion of SA Performance

5.5.1 Reliability

The reliability of the obtained solution is another important issue. Since the exact optimal solution to this problem is not known (note that no existing methods guarantee finding the global optimum), it is difficult to prove the goodness of the solution found by the proposed Simulated Annealing Algorithm. Therefore, an experiment is designed to statistically test the effectiveness of the algorithm. In this experiment the fitness value is evaluated for each randomly generated solution to the problem. First numerous solution samples are generated and tested. The next step is to fit a distribution to the fitness values for the random sample. Since the sample is randomly generated, the fitted distribution should approach the actual distribution of the fitness values for all possible solutions in the search space. Based on the distribution, we can compare the solution found by the SA algorithm and calculate the cumulative probability of the solution in the distribution. A lower cumulative probability indicates that most solutions in the search end up with a lower objective value than the one found by the SA algorithm. The lower the probability, the better the solution.

We first create random sample of 1,000,000 observations. The best fitness value in this sample is $3.7527*10^{9}$, while the worst one is $-1.0278*10^{12}$. The sample mean is $-2.2814*10^{11}$ and the standard deviation is $1.4481*10^{11}$, as shown in Figure 5.18. In the experiment, the optimized solution obtained ($4.0591*10^{9}$) is much higher than the highest value in the random sample ($3.7527*10^{9}$). In other words, the solution found by the SA algorithm dominates all other solutions in the distribution. In fact, the random sample does not cover the range of the fitness values for all possible solutions in the search space. The number of possible solutions for the unconstrained case is 27^{29} which includes unfeasible solutions. This number comes from the solution vector which has 30 elements. Besides the base year (year 1), in each year the number of stations in service can change from 4 to 30, so there are 27^{29} different permutations. It is difficult to calculate the exact number of possible feasible solutions, except writing a computer program to do it. For constrained cases, it is also difficult to calculate all the possible feasible solutions since the problem is dynamic. This suggests that the chosen sample size may not be large enough.

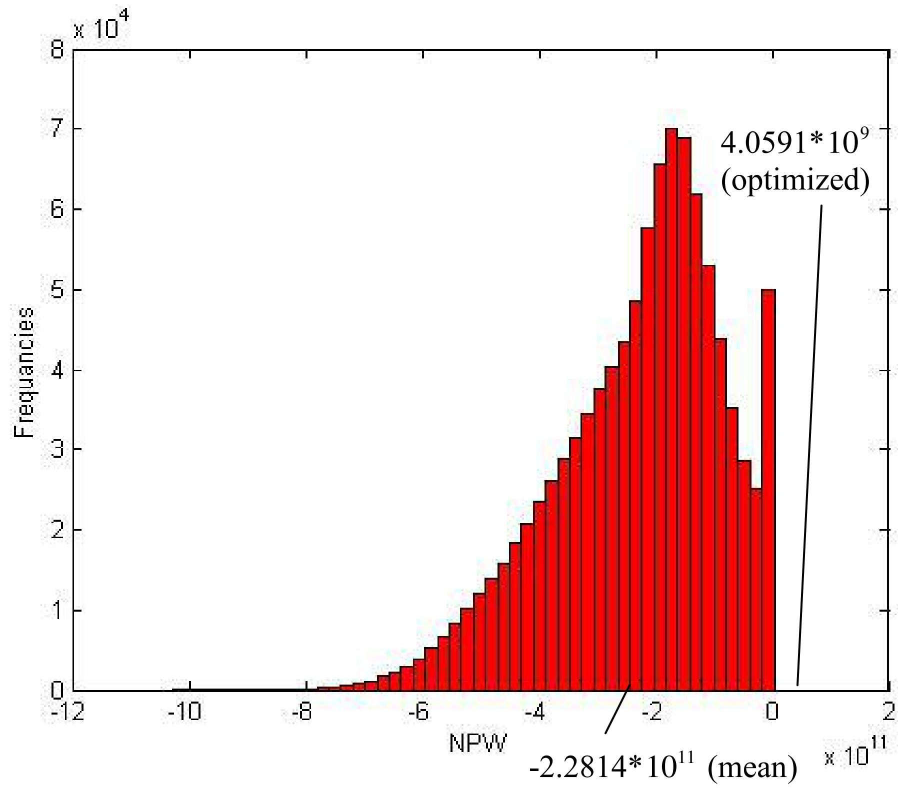

Fig. 5.18 Statistical Test

The best fitting distribution among those searched is the Extreme Value Distribution (EVD) as shown in Equation 5-1 and Figure 5.19.

$$y = f(x \mid \mu, \sigma) = \sigma^{-1}\left(\frac{x-\mu}{\sigma}\right)\exp\left(-\exp\left(\frac{x-\mu}{\sigma}\right)\right) \tag{5-1}$$

where μ = -1.61*10^{11}, standard error = 1.24*10^{8}

σ = -1.78* 10^{11}, standard error = 9.22*10^{7}

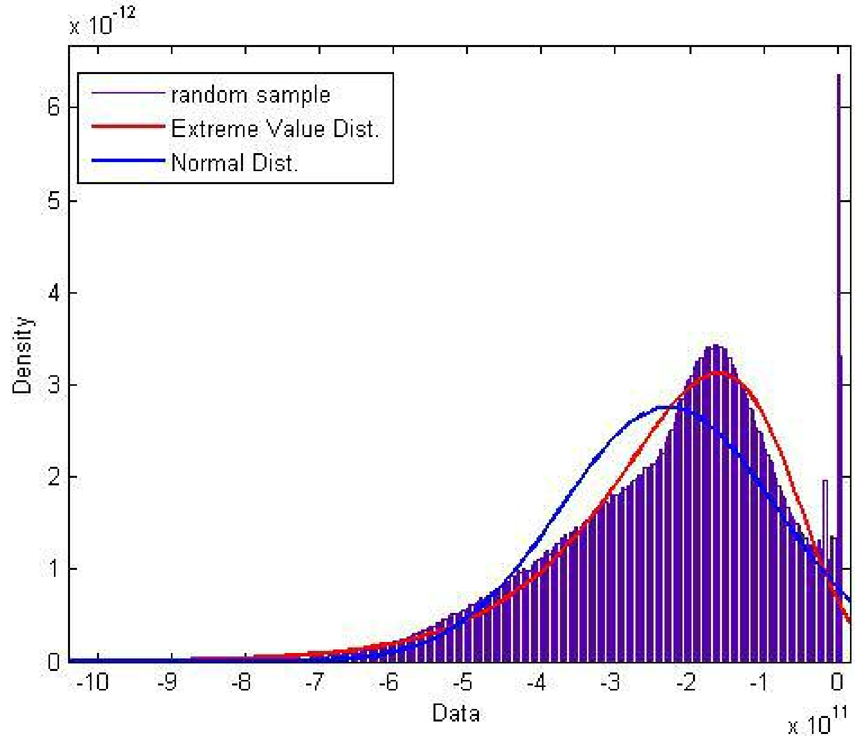

Fig. 5.19 The Fitted Extreme Value Distribution and Normal Distribution

In order to calculate the cumulative probability of the best solution found by the SA algorithm, the sample is fitted with a normal distribution based on its mean and standard deviation. The cumulative probability of the best solution ($4.0591*10^9$) found by the algorithm in the above normal distribution is P($f(x) \geq 4.0591*10^9$) = 0.0544. In other words, the solution ($4.0591*10^9$) dominates 94.56% of the solutions in the distribution. Since the sample is not normally distributed, the probability estimated might be wrong. However, the optimized solution value is better than 1 million random samples. The result shows that the best solution found by the SA algorithm, although not necessarily optimal, is still remarkably good when compared with other possible solutions in the search space. This analysis indicates a promising performance for the proposed optimization model.

5.5.2 Computation Time

One of the main drawbacks of the Simulated Annealing approach is its the computation time. As the problem size changes from ten stations and ten years to thirty stations and thirty years, the computation time increases significantly, as shown in Figure 5.20. Various computations such as computation of the net present worth function and computation of the probability of accepting bad solutions increase the computation time when the problem size grows. Also, for better results the cooling schedule has to be carried out very slowly and this significantly increases the computation time.

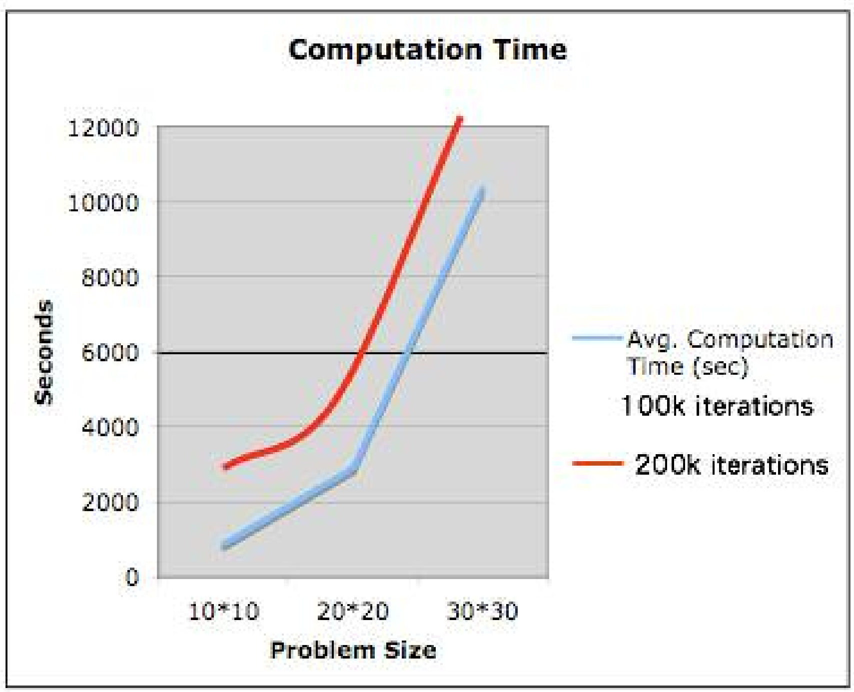

Fig. 5.20 Computation Time

Chapter 6: Sensitivity Analysis

The following sensitivity analyses are designed to investigate the effects of various input parameters (i.e., interest rate, fare taxation, value of time, operating cost, and demand growth rate) on the resulting optimized values (i.e., construction phases and total net benefits). These analyses show how sensitive the solutions of this work are to the values of input parameters. If the model is very sensitive to changes in a particular input parameter, that parameter should be predicted as accurately as possible and decisions should be made more cautiously.

6-1 Effects of Interest Rates (s)

The interest rate plays an important role in project scheduling, especially in large investment projects. Theoretically, projects tend to be postponed when the interest rate is high. If interest rate increases, then investment decreases due to the higher cost of borrowing. Interest rates are generally determined by the market, not by the government intervention. Although we cannot control the interest rate, sensitivity analysis for interest rate indicates how the extension decision changes with different interest rates.

To evaluate the effects of different interest rates (s) on phase decisions and the net present worth of total benefits (NPW), in this section s is varied between 0% and 30% while the base level is 5%. Table 6-1 shows the differences in optimized values and phases.

Table 6.1 Effects of Interest Rates on NPW and Optimized Phases

Interest rate	Cumulative net	#. phases	#. Stations in service in each year																													
(%)	benefits ($)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	1.20E+10	8	4	5	7	7	9	10	10	10	11	11	13	13	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
3	6.19E+09	8	4	5	7	7	9	10	10	10	11	11	13	13	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
5	4.05E+09	6	4	4	7	7	9	10	10	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
7	2.70E+09	6	4	5	7	7	9	10	10	10	11	11	11	11	11	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
10	1.53E+09	4	4	4	4	8	9	9	9	9	9	9	9	9	9	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
15	7.07E+08	2	4	4	4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
20	4.12E+08	3	4	4	4	5	5	5	5	5	5	5	5	5	5	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
30	2.39E+08	2	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Not only is the extension postponed but also the number of phases decreases when the interest rate increases. When the interest rate is below 10%, the transit route is extended to link 15. When the interest rate increases to 15%, the route is extended to link 8. When the interest rate exceeds 30%, the transit route merely extends to link 5. For links with enough demand, there is no problem delaying the construction. The marginal benefits of adding links with enough demand are always positive, except for adding links in the last year of the analysis period. However, links with low demand and high growth rate after extensions must be built earlier to make them beneficial over the analysis period. In order to achieve higher cumulative net benefits, the links which are possibly economically beneficial must be added as soon as possible. If some constraints prevent the extensions at early stages, the route cannot be extended as far as that in the unconstrained case.

Figure 6.1 shows the effects of interest rates (s) on the total net benefits (NPW). As s increases from 0% to 10%, NPW decreases rapidly. The NPW decreases slowly while s keeps increasing from 10% to 30%, since there are fewer additions of links.

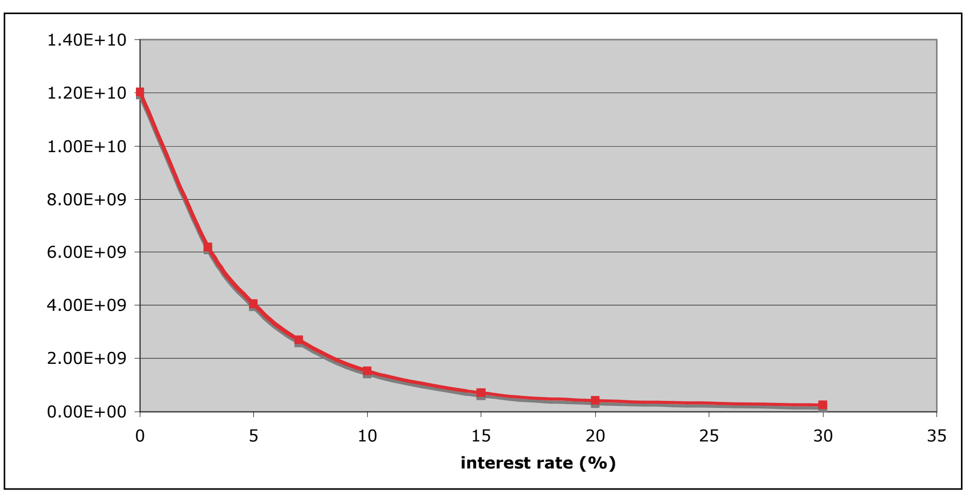

Fig. 6.1 Effects of Interest Rates on NPW

6-2 Effects of Taxation Ratios

The taxation of fare collection is one of the key parameters in this analysis of phased development. Taxation is used here to fund both operations and new investments. The taxation ratio is a policy decided by transit agencies. To evaluate the effects of different taxation ratios on the net present worth (NPW), in this section taxation ratios for funding operational expenses from 50% to 100% are considered. The base level is 70%. Table 6-2 shows the differences in optimized values and phases.

When the ratio decreases from 100% to 75%, NPW slightly increases. NPW decreases as the ratio ranges from 75% to 50%. As discussed in Chapter 5-5, revenue constraints are the dominant constraints that bind the results. Therefore, when the funds for operation decrease below 55%, NPW becomes negative, as shown in Figure 6.2. The negative number is due to the penalty applied in the model. Even with no extension, the funds cannot cover all operation expenses.

As the taxation ratio for operation decreases, the length of the transit route also decreases. The reason has been discussed before. Since some links with low demand cannot be added at early stages due to revenue constraints, adding those links is economically unbeneficial.

In Table 6.2, a taxation ratio between 75% and 80% achieves the highest NPW. Such analysis would help transit operators determine their best taxation policy for maximizing NPW.

Table 6.2 Effects of Taxation Ratios on *NPW* and Optimized Phases

Taxation of fare collection		Cumulative	#. Stations in service in each year																													
Investment	Operation	net benefits	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	100	4.20E+09	4	4	7	8	9	9	9	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
10	90	4.23E+09	4	4	7	8	9	10	11	11	11	11	11	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
15	85	4.29E+09	4	4	7	8	9	10	11	11	11	11	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
20	80*	4.30E+09	4	4	7	8	9	10	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
25	75*	4.30E+09	4	4	7	8	9	10	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
27	73	4.24E+09	4	4	7	8	9	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
30	70	4.06E+09	4	4	7	7	9	10	10	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
40	60	3.08E+09	4	4	4	5	5	5	7	7	7	8	8	8	9	9	10	10	11	11	11	11	11	11	11	11	11	11	11	11	11	11
45	55	1.36E+09	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6	6	9	9	9	9	9	9	9	9	9	9	9	9	9	9
50	50	-4.62E+09	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6	7	7	8	8	8	8	8	8	8	8	8	8

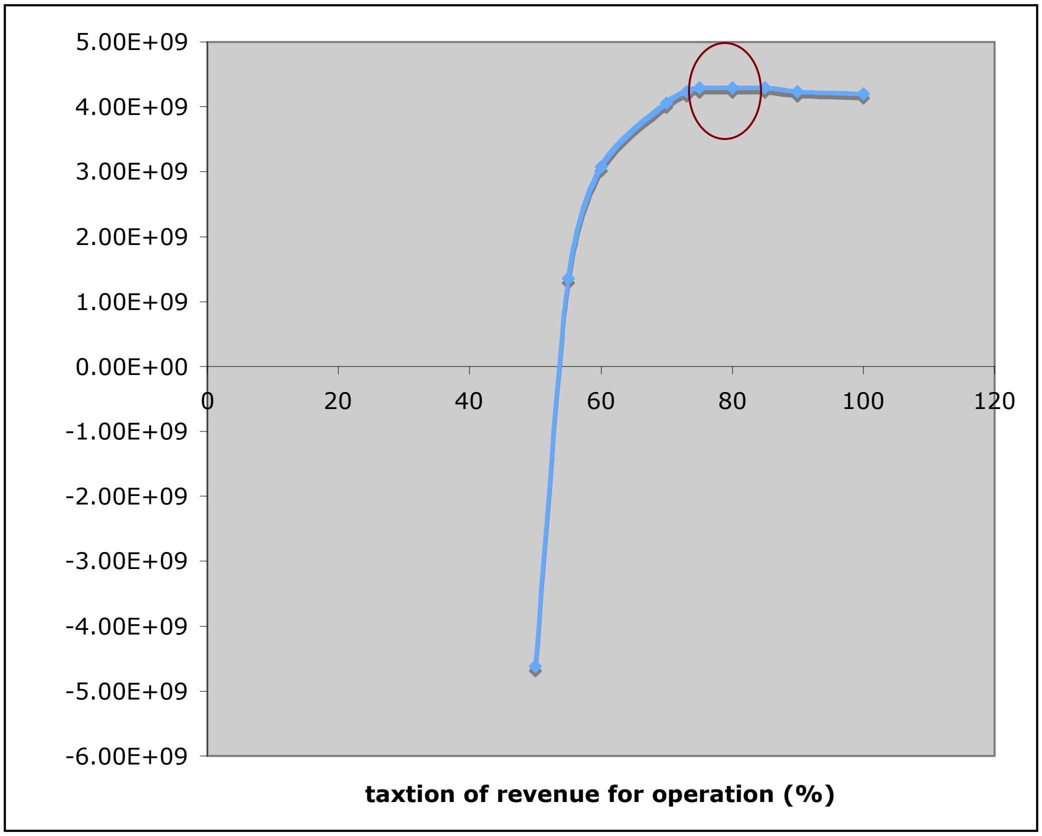

Fig. 6.2 Effects of Taxation Ratios on *NPW*

<u>*6-3 Effects of In-Vehicle Time Values*</u>

This section shows the effects of the value of user in-vehicle time (**Error! Objects cannot be created from editing field codes.**). Table 6.3 summarizes the results of sensitivity analysis with respect to the **Error! Objects cannot be created from editing field codes.** value. Values of in-vehicle time ($/passenger-hour) between 1 and 60 are analyzed, while the base level is 5.

Table 6.3 Effects of In-veh. Time Values on *NPW* and Optimized Phases

value of in-veh. time ($/pass-hr)	cumulative net benefits	#. Stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	5.22E+09	4	5	5	8	9	10	10	10	11	11	13	13	14	15	15	15	16	16	16	16	16	16	16	16	16	16	16	16	16	16
3	4.69E+09	4	4	7	7	9	9	10	10	11	11	11	11	11	15	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
5	4.05E+09	4	4	7	7	9	10	10	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
7	3.44E+09	4	5	5	8	9	9	9	9	11	11	11	11	11	11	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
10	2.86E+09	4	4	7	7	9	9	9	9	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
12	2.52E+09	4	4	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
15	2.11E+09	4	4	7	7	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
17	1.85E+09	4	4	4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
20	1.50E+09	4	4	4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
25	1.14E+09	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
30	9.06E+08	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
35	6.80E+08	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
40	4.53E+08	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
50	1.65E+05	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
60	-4.53E+08	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

As shown in Figure 6.3, the net present worth (*NPW*) decreases as **Error! Objects cannot be created from editing field codes.** ($/pass.-hr) increases. The slope of *NPW* is much steeper when **Error! Objects cannot be created from editing field codes.** is below 20. When **Error! Objects cannot be created from editing field codes.** exceeds 20, the slope of *NPW* becomes gradual. *NPW* becomes negative as **Error! Objects cannot be created from editing field codes.** exceeds 50. A shorter transit route can be expected as **Error! Objects cannot be created from editing field codes.** increases. The greater the **Error! Objects cannot be created from editing field codes.**, the shorter the transit route, since the in-vehicle costs increase as well as the total costs, while user benefits stay the same. Therefore, the marginal net benefits of adding one link decrease.

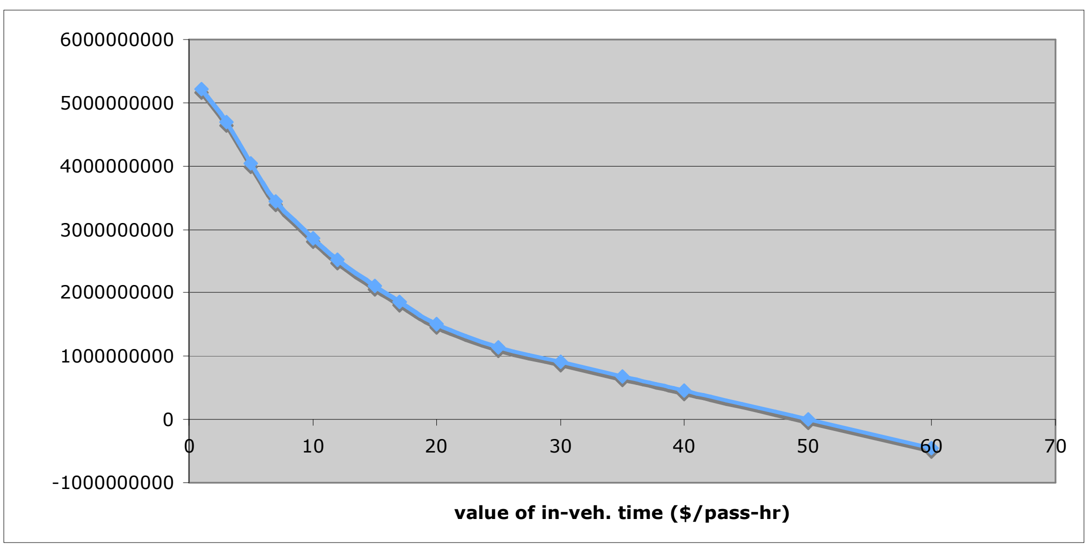

Fig. 6.3 Effects of In-veh. Time Values on *NPW*

6-4 Effects of User Waiting Time Values

The effects of different values of user waiting time (u_W) are examined. Table 6.4 summarizes the optimized results for different values of user waiting time. u_W ($/pass.-hr) values between 1 and 22 are tested, while the base level is 10.

Table 6.4 Effects of Waiting Time Values on *NPW* and Optimized Phases

value of waiting time ($/pass.-hr)	cumulative net benefits	#. Stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	5.52E+09	4	4	7	8	9	10	11	11	11	11	16	16	16	16	16	16	26	26	26	26	26	26	26	26	26	26	26	26	26	26
3	4.98E+09	4	4	7	8	9	10	11	11	11	11	16	16	16	16	16	16	26	26	26	26	26	26	26	26	26	26	26	26	26	26
5	4.77E+09	4	4	7	8	9	10	11	11	11	11	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
10	4.05E+09	4	4	7	7	9	10	10	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
15	2.59E+09	4	5	5	5	6	6	6	7	7	8	8	9	9	10	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
17	4.93E+08	4	4	4	4	4	4	4	4	5	5	5	5	9	9	9	9	9	9	11	11	11	11	11	11	11	11	11	11	11	11
20	2.08E+08	4	4	4	4	4	4	4	4	4	5	5	5	5	6	8	8	8	8	9	9	9	9	9	9	9	9	9	9	9	9
22	-1.36E+09	4	4	4	4	4	4	4	4	5	5	5	6	6	6	6	6	6	8	8	9	9	9	9	9	9	9	9	9	9	9

As shown in Figure 6.4, the net present worth (NPW) decreases as e_W (\$/pass.-hr) increases. NPW decreases slowly when e_W below 15, and drops significantly when e_W is between 15 and 17. The slope of NPW becomes flat when e_W is between 17 and 20. NPW becomes negative when e_W exceeds 20.

As with values of in-vehicle time, the greater the e_W, the shorter the transit route. However, the length of the transit route is more sensitive to user waiting time than to user in-vehicle time. When e_W decreases below 3, the transit route extends to link 26. In the previous section, the furthest point that the route can reach is link 16. The range of effective waiting time value is smaller than that of effective in-vehicle time value. This indicates that the user waiting time value has to be chosen more carefully, since it affects the solution significantly. Generally speaking, a high e_W results in construction postponements and fewer construction phases.

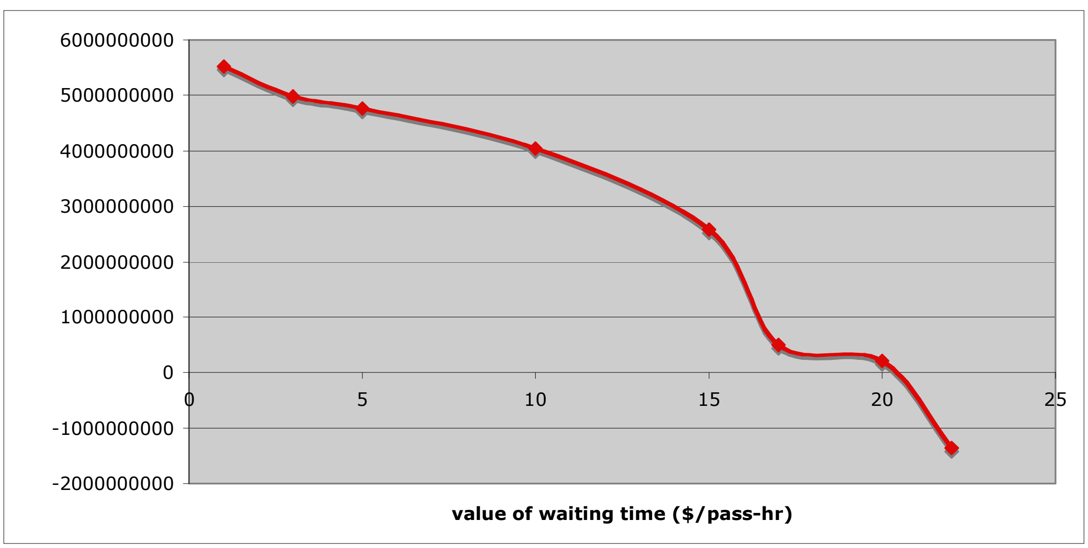

Fig. 6.4 Effects of Waiting Time Values on NPW

6-5 Effects of Hourly Operating Costs

The effects of different hourly operating costs (u_T) are examined. In this section u_T (\$/car-hr) values between 100 and 700 are considered, while the base level is 350. Table 6.5 summarizes the results.

The slope of the net present worth (NPW) decreases when hourly operating costs (u_T) increase except in the u_T interval between 250 and 300, as shown in Figure 6.5. For u_T values from 250 to 300, NPW slightly increases. NPW becomes negative as u_T exceeds approximately 660.

Table 6.5 Effects of Operating Costs on *NPW* and Optimized Phases

operating cost ($/car-hr)	cumulative net benefits	#. Stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
100	4.91E+09	4	4	7	8	9	10	10	10	10	15	16	16	16	16	16	16	16	26	26	26	26	26	26	26	26	26	26	26	26	26
150	4.77E+09	4	4	7	8	9	10	11	11	11	11	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
200	4.60E+09	4	4	7	8	9	10	11	11	11	11	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
250	4.03E+09	4	4	7	8	9	10	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
300	4.05E+09	4	4	7	7	9	10	10	10	11	11	11	11	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
400	3.45E+09	4	4	6	6	8	8	8	8	9	9	11	11	11	11	11	11	11	11	15	15	15	15	15	15	15	15	15	15	15	15
500	2.22E+09	4	4	4	4	4	4	4	6	6	6	8	8	8	8	9	9	11	11	11	11	11	11	11	11	11	11	11	11	11	11
600	2.13E+09	4	4	4	4	4	4	4	4	4	4	4	6	6	6	8	8	8	8	9	9	9	9	9	9	9	9	9	9	9	9
700	-1.27E+09	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	6	6	6	8	8	8	8	8	8	8	8	8	8	8	8

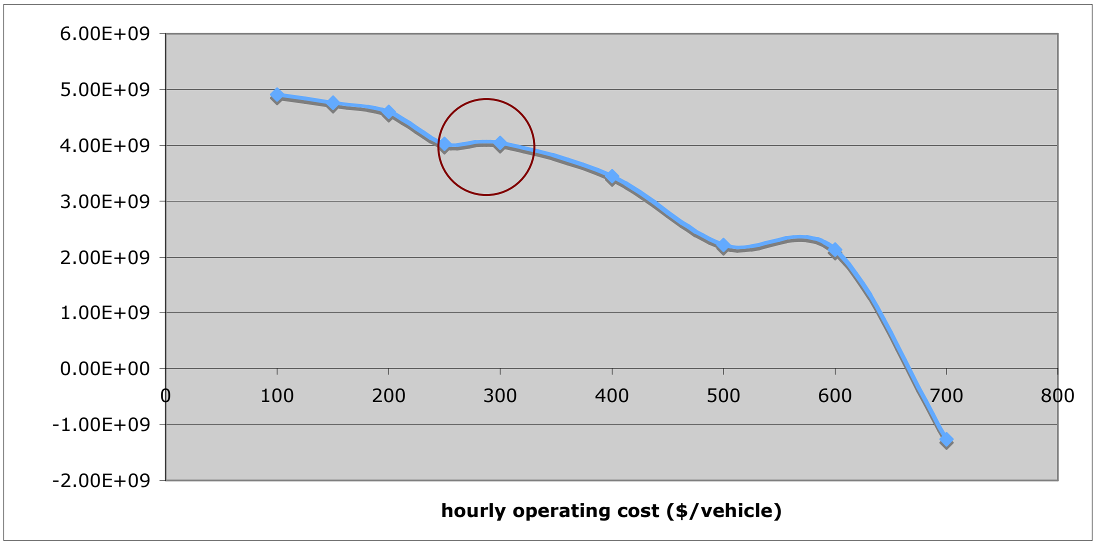

Fig. 6.5 Effects of Hourly Operating Costs on *NPW*

As shown in the previous sections, increasing the value of hourly operating costs shortens the route, since the operating costs increase as well as the total costs, while user benefits stay the same. Therefore, the marginal net benefits of adding one link decrease. In addition, increasing u_T reduces the number of construction phases and delays the extensions.

6-6 Effects of Demand Growth Rates

The demand growth rates (r) after extensions are assumed higher in rural areas than in the CBD. The effects of different demand growth rates are examined in this section. Table 6.6 summarizes the results.

Table 6.6 Effects of Demand Growth Rates on *NPW* and Optimized Phases

Demand Growth Rates (r)		*NPW* (\$)	Optimized Solution
before extensions	after extensions		
0%	5%	2.9280E+09	[4 5 7 8 9 9 11 11….11]
3%	0%	1.1130E+09	[4 4 4 4 4 4 4 9 9 …9]
3%	3%	2.0109E+09	[4 4 4 4 4 9 9 …9]
3%	5%	3.0855E+09	[4 4 7 8 9 9 11 11 …11]
3%	10%	8.9612E+09	[4 5 7 8 9 9 11 11 11 15 15 …15]
3%	15%	2.3668E+10	[4 5 7 8 9 10 11 12 13 15 16 16 16 16 16 26 26 …26]

In general, the higher demand growth rate speeds up the extensions and lengthens the transit route as well. When 0% growth rate after extensions is chosen, the links are added only if they have enough demand. When 0% growth rate before extensions and 5% after extensions are chosen, the addition of links with enough demand is limited by the revenue constraints. There is no construction at later stages due to the higher capital costs of adding links. However the demand growth rate changes, the model can determine the transit route length based on the potential demand and the growth rate.

Chapter 7: Conclusions

7-1 Summary of Research Results

The primary contributions of this research include:

(1) Development of an optimization model that maximizes the net present worth of total benefits under various financial situations for phased development of rail transit routes.

(2) Use of a Simulated Annealing Algorithm that obtains a near-optimal solution for this rail transit phased optimization problem.

(3) Analyses and comparisons for the effects of various financial constraints on the optimized phases and objectives based on assumed input parameters.

(4) Sensitivity analyses of optimized results (i.e. optimized phases and objectives) with respect to various input parameters.

7-2 Conclusions

The model has been developed for optimizing the construction phases for a rail transit extension project. It has been used to determine not only the construction phases but also the economic feasibility of additional links under various financial constraints. The optimized solution also avoids overextension of the proposed route. In addition, tax-funding policy also can be optimized through sensitivity analysis, as demonstrated.

Based on the numerical examples, the thesis leads to the following conclusions:

(1) The numerical analyses show that for the unconstrained case, immediately adding all links with positive net benefits achieves the highest objective value. This result is consistent with the one found in Kolisch and Padman (2001), which is to schedule jobs with positive cash flows as soon as possible and to delay jobs with negative cash flows as much as possible. There is only one phase (no delay) in this problem since no completion constraint is applied. Therefore, those links with negative values are postponed indefinitely. Delaying extensions when demand is low does not occur in the problem with the objective of maximizing net benefits, but it probably appears in the problem with the objective of maximizing profits or minimizing costs (with completion constraints).

(2) For the case with higher demand growth rate after extension, the economic feasibility of adding one link is affected significantly by the construction time. Compared with various financial constraints, the transit route can be extended to link 27 for the unconstrained case, but it can only be extended to link 15 for the constrained case. If some links with low demand and high growth rate after extension cannot be added at early stages, they would no longer be economically beneficial. That is due to the high capital costs of adding links. In our sensitivity analyses, no extension occurred at later stages validates this. Consequently, when

analyzing high capital cost project's economic feasibility, construction phases should be taken into account.

(3) The sensitivity analysis is conducted to obtain more accurate input parameter effects on the optimized phases. Several parameters are tested, including interest rates (s), taxation ratios for construction investment, values of user in-vehicle time (u_I), values of user waiting time (Error! Objects cannot be created from editing field codes.), and hourly operating costs (u_T). In general, when the values of these variables increase, we find decreases in the net present worth (NPW), as well as the transit route and the number of the construction phases. In addition, NPW is maximized when the taxation ratio for investment is between 20% and 25%.

7-3 Recommendations Further Research

The following extensions are suggested for further research:

(1) The model designed in this thesis is deterministic. Based on uncertainties about the future, this model can be improved to a probabilistic version. For instance, the demand growth rate might change over time. Demand will not necessarily increase in the future. Interest rates and inflation rates also vary over time. A probabilistic model can address this problem more realistically than a deterministic model.

(2) For more realistic reasons, future model should consider relaxing the simplifying assumptions, such as no binding construction time constraint

and adding links sequentially. Currently the model can be used in cases with radial network. If the demand is not radially distributed, the assumption that adds links sequentially should be relaxed.

(3) Many variables that would be affected by transit extension can be considered in this model, such as the inflation rate, and the life cycle of the rolling stock. External benefits and costs can be added into the model if they are correctly estimated, including employment opportunities, land values, travel time saving, and environmental impacts.

(4) Some operational variables (e.g. transit fare and cruise speed) can also be optimized by a modified model. In a real operation, transit agencies do not often adjust the fare, but fixing fare for over 30-year horizon is assumed in this thesis. In order to optimize these variables, price and travel time elasticity of the demand must be considered.

(5) This model optimizes the construction phases for single one route. If the network has branches or multiple routes, the problem becomes more complex, and this model cannot yet deal with such complex networks. It should be improved to handle more complex networks in the future.

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